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Particular solutions of the equations for the lowest Green's functions in the infrared region of quantum chromodynamics

B. A. Arbuzov and A. I. Davydychev

Institute for High Energy Physics,
Serpukhov, Moscow Region, USSR

Abstract

The solutions of the Schwinger-Dyson equations for the ghost and quark propagators in the infrared region of quantum chromodynamics are studied. It is shown that there exists a gauge in which the equation for the ghost propagator has the free solution. This result is used to justify the possibility of omitting the contributions of the ghosts from the gauge identity for the quark-gluon vertex in this distinguished gauge. Under this condition, a new class of solutions to the equation for the quark propagator is obtained; the class includes solutions with broken chiral invariance that are manifestly nonanalytic with respect to the coupling constant.

1 Introduction

The infrared region of QCD, which is extremely complicated for theoretical analysis because of the inapplicability of perturbation theory, is very important for a number of problems associated with strong interactions at large distances. Of particular interest is the infrared behavior of the lowest Green's functions – the gluon, quark, and ghost propagators, vertex functions, etc. In some studies, the problem of color confinement has been associated with the infrared asymptotic behavior of the gluon propagator [1], and also with the analytic properties of the quark propagator [2, 3].

Our point of departure for studying the infrared region of QCD in the present paper is based on asymptotic solution of the system of Schwinger–Dyson equations with allowance for the gauge identities [4, 5]. The use of this approach shows that the Schwinger–Dyson equation for the gluon propagator $D_{\mu\nu}(k)$ (in the axial gauge) is satisfied by the infrared asymptotic behavior $D(k) \sim M^2/(k^2)^2$. It is important to emphasize that this behavior can be realized only when the three-gluon vertex function has a definite form [5], consistent, of course, with the corresponding Ward–Slavnov–Taylor identity [6]. The gluon propagator and the vertex of the required form are described by the following effective Lagrangian in the infrared region [5]:

$$\mathcal{L}_{\rm eff} = \frac{1}{4M^2}\, D^{ab}_{\rho} F^{b}_{\mu\nu}\, D^{ac}_{\rho} F^{c}_{\mu\nu} - \frac{g}{6M^2}\, f^{abc}\, F^{a}_{\mu\nu}\, F^{b}_{\nu\rho}\, F^{c}_{\rho\mu}\ , \tag{1}$$

where $F^{a}_{\mu\nu}$ is the intensity of the gluon field, D^{ab}_{ρ} is the covariant derivative defined in the usual manner, and f^{abc} are the structure constants of the gauge group (in our case SU(3)). The Lagrangian (1) is manifestly gauge invariant.

In this paper, using the infrared behavior of the gluon propagator that follows from (1), we consider the solutions of the Schwinger–Dyson equations for the ghost and quark propagators. The equation for the quark propagator has already been investigated in a number of studies [7, 8, 9, 10]. Here, we shall use a covariant gauge, i.e., a gluon propagator of the form

$$D^{ab}_{\mu\nu}(k) = \delta^{ab}\, \frac{M^2}{(k^2)^2} \left(g_{\mu\nu} - d\, \frac{k_\mu k_\nu}{k^2} \right) \equiv \delta^{ab}\, D_{\mu\nu}(k)\ . \tag{2}$$

In this gauge, the Faddeev–Popov ghosts greatly complicate the situation. Nevertheless, it is simpler to work in this gauge than in the ghostless axial gauge. In the covariant gauge, the choice of the gauge parameter d and the related question of the contribution of the ghosts are decisive. Therefore, we begin by studying the infrared behavior of the lowest ghost Green's functions.

2 Green's functions for the ghosts

We consider first the choice of the gauge condition and the corresponding addition to the effective Lagrangian (1) that would fix the gauge and correspond to a singular structure of the gluon propagator (2). It is easy to show that the gauge condition can be taken in the form

$$\Phi^a = \Box^{1/2} \partial_\rho A^a_\rho = 0\ , \tag{3}$$

(where $\partial_\rho \equiv \partial/\partial x_\rho$, $\Box \equiv \partial_\mu \partial_\mu$), and the corresponding gauge-fixing term in the Lagrangian can be represented in the form

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\alpha M^2} \left(\Box^{1/2}\partial_\rho A^a_\rho\right)^2 = -\frac{1}{2\alpha M^2} \left(\partial_\rho A^a_\rho\right)\Box\left(\partial_\rho A^a_\rho\right) \ , \tag{4}$$

where $\alpha = 1 - d$. The gauge (3) belongs to the class of generalized α gauges [11].

Further, following the usual rules [11], we construct the ghost operator

$$M^{ab} = \delta^{ab}\Box^{3/2} + f^{abc}\Box^{1/2}\partial_\rho A^c_\rho = \Box^{1/2}\partial_\rho D^{ab}_\rho \ , \tag{5}$$

and also the Lagrangian of the Faddeev–Popov ghosts:

$$\mathcal{L}_{\rm gh} = \bar{c}^a(x)\left(\partial_\rho D^{ab}_\rho\right)c^b(x) \ . \tag{6}$$

To this Lagrangian there correspond the following ghost propagator and ghost-ghost-gluon vertex:

$$S^{ab}(k) = \delta^{ab}(k^2)^{-3/2} \ , \qquad \Lambda^{abc}_\mu(k,p;p-k) = f^{abc}k_\mu(k^2)^{1/2} \ . \tag{7}$$

The lowest Green's functions for the ghosts (7) satisfy the condition of gauge invariance, since the sum of the Lagrangians (1), (4), and (6) is by construction invariant with respect to the well-known transformations [12]

$$\begin{array}{rclrcl} A^a_\mu(x) & \to & A^a_\mu + \delta A^a_\mu \ , & \delta A^a_\mu & = & D^{ab}_\mu c^b(x)\cdot\delta\zeta \ ; \\ c^a(x) & \to & c^a + \delta c^a \ , & \delta c^a & = & -\frac{1}{2}f^{abc}c^b(x)c^c(x)\cdot\delta\zeta \ ; \\ \bar{c}^a(x) & \to & \bar{c}^a + \delta\bar{c}^a \ , & \delta\bar{c}^a & = & -\dfrac{1}{\alpha M^2}\Box^{1/2}\,\partial_\rho A^a_\rho\cdot\delta\zeta \ . \end{array} \tag{8}$$

where $(\delta\zeta)^2 = 0$, $\delta\zeta\cdot c^a + c^a\cdot\delta\zeta = 0$, $\delta\zeta\cdot\bar{c}^a + \bar{c}^a\cdot\delta\zeta = 0$, $[\delta\zeta, A^a_\mu] = 0$

Note that when the Feynman diagrams containing ghosts are calculated by means of the expressions (7) the contribution of the ghost loops is exactly the same as in the ordinary Lorentz gauge[1].

We must now discuss the actual choice of the gauge parameter d in the expression (2). In various studies [7, 10, 13] made by means of dimensional regularization (space-time dimension $n = 4 + 2\varepsilon$) it has been shown that the most convenient choice of the gauge parameter from the point of view of canceling of the gauge divergences is

$$d = \frac{4}{5-n} = \frac{4}{1-2\varepsilon} \ . \tag{9}$$

In this case, the gluon propagator (2) has the remarkable property of transversality in the coordinate space (moreover, it does not contain poles with respect to ε):

$$D^{ab}_{\mu\nu}(x) = \delta^{ab}M^2 {\rm i}^{-1-2\varepsilon}\pi^{-2-\varepsilon}\,\frac{\Gamma(1+\varepsilon)}{8(1-2\varepsilon)}\,(x^2)^\varepsilon\left(g_{\mu\nu} - \frac{x_\mu x_\nu}{x^2}\right) , \qquad D^{ab}_{\mu\nu}(x)x_\nu = 0 \ . \tag{10}$$

Therefore, in a certain sense the gauge (9) is analogous to the well-known Soloviev–Yennie gauge in electrodynamics [14]. In this paper, we shall show that the use of the gauge (9) leads to interesting consequences for the lowest ghost Green's functions.

[1]This circumstance can also be understood by removing from Det$M$ the field-independent factor Det $\Box^{1/2}$; then the action for the ghosts in the functional integral will have the standard form.

We consider the Schwinger–Dyson equation for the ghost propagator [15] (to simplify the expressions, we shall omit the color indices)

$$S^{-1}(p) = S^{(0)^{-1}}(p) + \frac{g^2\, C_2}{(2\pi)^n \mathrm{i}} \int \mathrm{d}^n k\; D_{\mu\nu}(k)\; \Lambda^{(0)}_\mu(p-k, p; k)\; S(p-k)\; \Lambda_\nu(p, p-k; -k)\;, \quad (11)$$

where C_2 is a color factor (for the group $\mathrm{SU}(N)$, $C_2 = N$). If in Eq. (11) we substitute the expressions (7) as the total and free Green's functions for the ghosts, then the loop integral will be equal to zero, and the equation will be satisfied exactly. Thus, we have obtained an exact solution of Eq. (11) consistent with the requirements of gauge invariance. This solution was obtained in the gauge (9) (in which the gluon propagator is transverse in the coordinate space), and this is one further factor making such a gauge convenient. We note that in [16] such a solution was not obtained for precisely the reason that a different gauge was used in that paper. The question of the uniqueness of the obtained solution remains open.

Thus, if the solution that we have found to Eq. (11) is realized, this indicates that the ghost sector of the QCD Lagrangian is not changed (in the gauge (9) on the transition from the bare Lagrangian to the effective Lagrangian in the infrared region.

The total ghost propagator is usually expressed in the form

$$S(k) = \frac{S^{(0)}(k)}{1 + b(k^2)}\;, \quad (12)$$

where $b(k^2)$ corresponds to the ghost self-energy and is proportional to the loop integral in Eq. (11). Therefore, realization of the solution (7) is equivalent to the result

$$b(k^2) = 0\;. \quad (13)$$

We shall now consider the Ward–Slavnov–Taylor identity for the quark-gluon vertex in the presence of ghosts [15]:

$$k_\mu\, \Gamma^a_\mu(p, q; k)\, \big(1 + b(k^2)\big) = H^a(k; p) G^{-1}(q) - G^{-1}(p) H^a(k; p)\;, \quad (14)$$

where the function $b(k^2)$ is determined by the relations (12)–(13) and is equal to zero, and $H^a(k; p) = t^a + \beta^a(k; p)$, where t^a are the generators of the gauge group. The structure $\beta^a(k; p)$ is proportional to a loop integral containing a strongly connected fourfold quark-quark-ghost-ghost vertex. The question of the infrared behavior of this structure requires, of course, a special study, but the vanishing of the loop term $b(k^2)$ gives grounds for hoping that the structure β too vanishes or, at least, does not have a significant influence on the obtained results. On the basis of these considerations, we shall work in the framework of the model of "Abelian chromodynamics" [17], in which one uses the "simplified" identity (14)

$$k_\mu\, \Gamma^a_\mu(p, q; k) = G^{-1}(q) - G^{-1}(p)\;, \qquad k + q - p \quad (15)$$

(the color indices are omitted), i.e., the Ward–Fradkin–Takahashi identity [18].

3 Solutions of the equation for the quark propagator

Before we turn to the investigation of the Schwinger–Dyson equation for the quark propagator, we obtain some auxiliary formulas.

We consider a loop integral of the form

$$\int \mathrm{d}^n k \cdot D_{\mu\nu}(p-k)\gamma_\mu\gamma_\nu f(k^2)\ , \tag{16}$$

where $D_{\mu\nu}(k)$ is the gluon propagator (2) in the gauge (9), and $f(k^2)$ is a scalar function. Going over to Fourier transforms in the coordinate space, and integrating over the momentum, we obtain

$$(2\pi)^n \int \mathrm{d}^n x \cdot e^{-\mathrm{i}(px)} D_{\mu\nu}(x)\gamma_\mu\gamma_\nu f(x^2)\ . \tag{17}$$

Using for $D_{\mu\nu}(x)$ the expression (10), we obtain

$$D_{\mu\nu}(x)\ \gamma_\mu\gamma_\nu = \mathrm{i}^{1--2\varepsilon}\pi^{-2-\varepsilon} M^2 \frac{(3+2\varepsilon)\ \Gamma(1+\varepsilon)}{8(1-2\varepsilon)}\ (x^2)^\varepsilon\ , \tag{18}$$

where $\varepsilon = (n-4)/2$. From this it can be seen that provided the Fourier transform $f(x^2)$ satisfies certain conditions (which will be specified below) we can go to the limit $n \to 4$ in the expression (17), and we then obtain the helpful formula

$$\lim_{n\to 4} \int \mathrm{d}^n k \cdot D_{\mu\nu}(p-k)\gamma_\mu\gamma_\nu f(k^2) = -6\mathrm{i}\pi^2 f(p^2) M^2. \tag{19}$$

One can show similarly that

$$\lim_{n\to 4} \int \mathrm{d}^n k \cdot D_{\mu\nu}(p-k)\gamma_\mu \not{k} \gamma_\nu f(k^2) = 6\mathrm{i}\pi^2 \not{p} f(p^2) M^2. \tag{20}$$

We consider the conditions of applicability of these expressions for the example of (19). The first of the conditions required for the passage to the limit $n \to 4$ is that $f(x^2)$ should not contain divergences as $\varepsilon \to 0$. The second condition is related to the specific properties of dimensional regularization, in accordance with which, for example, the integral of a polynomial is zero [19, 20]. Another manifestation of these properties is that the Fourier transform of the polynomial is also equal to zero, i.e., Fourier transformation in dimensional regularization is in a certain sense degenerate. Thus, a second condition of applicability of formula [19] is the absence in $f(k^2)$ of degenerate terms whose Fourier integral gives zero. Then under the inverse Fourier transformation we obtain the same function $f(k^2)$ from which the direct transformation was made.

All that we have said about formula [19] applies equally to [20], except that instead of $f(k^2)$ it is necessary to consider the Fourier transform of $\not{k} f(k^2)$. Thus, if the listed conditions are satisfied,

$$\lim_{n\to 4} \int \mathrm{d}^n k \cdot D_{\mu\nu}(p-k)\gamma_\mu \left[\not{k} f_1(k^2) + f_2(k^2)\right] \gamma_\nu = 6\mathrm{i}\pi^2 \left[\not{p} f_1(p^2) - f_2(p^2)\right] M^2. \tag{21}$$

We emphasize once more that this relation holds only in the gauge (9).

We now consider the Schwinger–Dyson equation for the quark propagator:

$$1 = (\not p - m_0)\, G(p) + \frac{g^2 C_F}{(2\pi)^n \mathrm{i}} \int \mathrm{d}^n k\; D_{\mu\nu}(p-k)\; \gamma_\mu\; G(k)\; \Gamma_\nu(k,p;p-k)\; G(p)\;, \qquad (22)$$

where m_0 is the bare mass of the quark, and C_F is the color factor (equal to $(N^2-1)/(2N)$ for the group SU(N); 4/3 for the group SU(3)). As condition on the vertex, we shall use the gauge identity in the form (15). This identity recovers the vertex function up to a structure that is transverse with respect to the momentum k:

$$\Gamma_\mu(p,q;k) = \Gamma^{(\mathrm{L})}_\mu(p,q;k) + \Gamma^{(\mathrm{T})}_\mu(p,q;k)\;; \qquad q-p-k=0\;.$$

We note that the separation into transverse and longitudinal parts is, in general, somewhat arbitrary. For the longitudinal part of the vertex, we use a representation equivalent to the one obtained in [21]:

$$G(p)\Gamma^{(\mathrm{L})}_\mu(p,q;q-p)G(q) = \frac{1}{q^2-p^2}\left[G(p)\big(\not p\gamma_\mu + \gamma_\mu \not q\big) - \big(\not p\gamma_\mu + \gamma_\mu \not q\big)G(q)\right]\;, \qquad (23)$$

whence

$$\Gamma^{(\mathrm{L})}_\mu(p,q;q-p) = \frac{1}{q^2-p^2}\left[\big(\not p\gamma_\mu + \gamma_\mu \not q\big)G^{-1}(q) - G^{-1}(p)\big(\not p\gamma_\mu + \gamma_\mu \not q\big)\right]\;. \qquad (24)$$

The vertex (24) satisfies, of course, the identity (15).

On the basis of the treatment given in [10], we choose the transverse part of the vertex function in the form

$$\Gamma^{(\mathrm{T})}_\mu(p,q;q-p) = 2f(p,q)\;(\not p + m_0)^{-1}\,(\not p\gamma_\mu\not q - \not q\gamma_\mu\not p)\;(\not q + m_0)^{-1}\;, \qquad (25)$$

where the matrix structure $(\not p\gamma_\mu\not q - \not q\gamma_\mu\not p)$ is, apart from a factor, identical to the structure used in [10], $[(\not p + \not q)\sigma_{\mu\nu}k_\nu + \sigma_{\mu\nu}k_\nu(\not p + \not q)]$, while $f(p,q)$ is an arbitrary scalar function symmetric with respect to p and q.

It is convenient to rewrite the quark propagator $G(p) = \widetilde{A}(p^2)\not p + \widetilde{B}(p^2)$ in the form

$$G(p) = \left(A(p^2) + B(p^2)\frac{\not p}{p^2}\right)(\not p + m_0)\;. \qquad (26)$$

Then

$$\widetilde{A}(p^2) = A(p^2) + \frac{m_0}{p^2}\,B(p^2)\;, \qquad \widetilde{B}(p^2) = m_0 A(p^2) + B(p^2)\;. \qquad (27)$$

Note that if $m_0 = 0$ then $\widetilde{A}$ and $\widetilde{B}$ are equal to A and B, respectively.

Substituting (23), (25), and (27) in (22), we arrive at the equation

$$1 = (p^2 - m_0^2)\left(A(p^2) + \frac{\not p}{p^2}\,B(p^2)\right) + \frac{g^2\,C_F}{(2\pi)^n\mathrm{i}}\left(I^{(\mathrm{L})}(p) + I^{(\mathrm{T})}(p)\right)\;, \qquad (28)$$

where in the limit $n \to 4$, using (21), we obtain

$$
\begin{aligned}
I^{(\mathrm{L})}(p) &= 6\mathrm{i}\pi^2 M^2 \left(A(p^2) + \frac{m_0}{p^2}\, B(p^2) \right) , \\
I^{(\mathrm{T})}(p) &= \frac{8M^2}{p^2} \left(A(p^2)\not{p} + B(p^2) \right) \lim_{n\to 4} \int \frac{\mathrm{d}^n k \cdot f(k,p)\, B(k^2)}{k^2((p-k)^2)^2} \left(p^2 k^2 - (pk)^2 \right) .
\end{aligned}
$$

Further, equating in Eq. (28) the expressions multiplying the various matrix structures and combining the obtained equations, we arrive at the system

$$
A(p^2)(p^2 - m_0^2 + \kappa^2) + \kappa^2 m_0 \frac{B(p^2)}{p^2} - (p^2 - m_0^2)\frac{B^2(p^2)}{p^2 A(p^2)} = 1 , \tag{29}
$$

$$
B(p^2)(p^2 - m_0^2) + \frac{4}{3}\, A(p^2)\frac{\kappa^2}{\mathrm{i}\pi^2} \lim_{n\to 4} \int \frac{\mathrm{d}^n k \cdot f(k,p)\, B(k^2)}{k^2((p-k)^2)^2} \left(p^2 k^2 - (pk)^2 \right) = 0 , \tag{30}
$$

where $\kappa^2 = 3g^2 M^2 C_F/(8\pi^2)$ (for the group SU(3), $\kappa^2 = g^2 M^2/(2\pi^2)$). The parameter κ^2 is related to the slope of the linearly increasing section of the quark-antiquark potential and is therefore given by experiment.

The case $B(p^2) = 0$ corresponds to a well-known solution of this system [10]:

$$
G(p) = \frac{\not{p} + m_0}{p^2 - m_0^2 + \kappa^2} , \tag{31}
$$

$$
\Gamma_\mu^{(\mathrm{L})}(p, q, q-p) = \gamma_\mu - \kappa^2 \, (\not{p} + m_0)^{-1} \gamma_\mu (\not{p} + m_0)^{-1} , \tag{32}
$$

$$
\Gamma_\mu^{(\mathrm{T})}(p, q, q-p) = 2f(p,q)\, (\not{p} + m_0)^{-1} \, (\not{p}\gamma_\mu \not{q} - \not{q}\gamma_\mu \not{p}) \, (\not{q} + m_0)^{-1} , \tag{33}
$$

where $f(p, q)$, as in (25), is an arbitrary symmetric function. Note that for $m_0 = 0$ the quark propagator does not violate chiral invariance:

$$
G(p) = \frac{\not{p}}{p^2 + \kappa^2} . \tag{34}
$$

To obtain other solutions of the system (29)–(30), we assume that $f(p, q)$ factorizes, i.e., $f(p, q) = \psi(p^2)\psi(q^2)$. Then from the integral equation (30), going over to four-dimensional Euclidean space, we can obtain [22] the differential equation ($y = -p^2$)

$$
\frac{\mathrm{d}}{\mathrm{d}y} \left[y^3 \, \frac{\mathrm{d}}{\mathrm{d}y} \left(\frac{(y + m_0^2)\, B(y)}{yA(y)\, \psi(y)} \right) \right] = 2\kappa^2 \, y\psi(y) B(y) . \tag{35}
$$

Equation (35) is equivalent to the integral equation (30) if the following boundary conditions are satisfied:

$$
\lim_{y\to 0} \left[y^3 \, \frac{\mathrm{d}}{\mathrm{d}y} \left(\frac{(y + m_0^2)\, B(y)}{y\, A(y)\, \psi(y)} \right) \right] = 0 , \qquad \lim_{y\to\infty} \left[\frac{1}{y} \, \frac{\mathrm{d}}{\mathrm{d}y} \left(\frac{y(y + m_0^2)\, B(y)}{A(y)\, \psi(y)} \right) \right] = 0 . \tag{36}
$$

The solution for the case $B(y) = 0$ is already known (Eqs. (31)–(33)). We shall seek solutions for which $B(y) \neq 0$. We introduce the function

$$
Y(y) = \frac{B(y)\sqrt{y + m_0^2}}{\sqrt{-y\, A(y)}} . \tag{37}
$$

In addition, we impose on $\psi(y)$ the condition

$$\psi(y)\sqrt{-y\,A(y)} = \frac{\Lambda^{1/2}}{\kappa}\,\sqrt{y+m_0^2}\;, \tag{38}$$

where $\Lambda = \text{const}$. The condition (38) enables us to obtain a class of solutions in explicit form. In the new notation, the differential equation (35) becomes linear and homogeneous and takes the form

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[y^3\,\frac{\mathrm{d}Y(y)}{\mathrm{d}y}\right] = -2\Lambda\,y\,Y(y)\;, \tag{39}$$

with the boundary conditions

$$\lim_{y\to 0}\left(y^3\,\frac{\mathrm{d}Y(y)}{\mathrm{d}y}\right) = 0\;, \qquad \lim_{y\to\infty}\left(\frac{1}{y}\,\frac{\mathrm{d}}{\mathrm{d}y}\,\left(y^2Y(y)\right)\right) = 0\;. \tag{40}$$

The complete solution of Eq. (39) has the power form

$$Y(y) = C\left(\frac{\kappa^2}{y}\right)^{1-a} + C'\left(\frac{\kappa^2}{y}\right)^{1+a}\;, \tag{41}$$

$$a = \sqrt{1-2\Lambda}\;, \tag{42}$$

where C and C' are arbitrary constants. In conjunction with the definition (42), the boundary conditions (40) in the case of real values of a give the restriction $0 \le a < 1$.

The first of the equations of the system (29) and the definition (37) can be used to express the functions A and B in terms of Y:

$$B(y) = \frac{\kappa^2 m_0 Y^2 \pm \sqrt{Y^2\left[4y(1-Y^2)(y+m_0^2)(y+m_0^2-\kappa^2)+m_0^2\kappa^4Y^2\right]}}{2(y+m_0^2)(y+m_0^2-\kappa^2)}$$

$$A(y) = -\frac{y+m_0^2}{y}\,\frac{B^2(y)}{Y^2(y)}\;. \tag{43}$$

At the same time, ψ is expressed in terms of A by means of formula (38). In the case of an arbitrary bare mass, the expressions (43) have a rather cumbersome form. For $m_0 = 0$, they simplify appreciably:

$$A(y) = -\frac{1-Y^2(y)}{y-\kappa^2}\;, \qquad B^2(y) = Y^2(y)\,\frac{1-Y^2(y)}{y-\kappa^2}\;. \tag{44}$$

The expressions (44), like (34), contain an "unphysical" denominator, which can, in principle, be removed by requiring $(C+C')^2 = 1$. In the set then obtained (for $m_0 = 0$), the solution with $a = 1/2$ $(\Lambda = 3/8)$ has the simplest form. It is easy to show that in this case the conditions of applicability of (19) and (20) require $C' = 0$. As a result, we obtain the following expressions for the corresponding Green's functions:

$$G(p) = \frac{\not p + \sigma\kappa}{p^2}\;, \tag{45}$$

$$\Gamma^{(\mathrm{L})}_\mu(p,q,q-p) = \gamma_\mu - \kappa^2\,(\not p+\sigma\kappa)^{-1}\gamma_\mu(\not p+\sigma\kappa)^{-1}\;, \tag{46}$$

$$\Gamma^{(\mathrm{T})}_\mu(p,q,q-p) = \frac{3}{4\kappa^2}\,\frac{\not p}{\sqrt{-p^2}}\,(\not p\gamma_\mu\not q - \not q\gamma_\mu\not p)\,\frac{\not q}{\sqrt{-q^2}}\;, \tag{47}$$

where $\sigma = \pm 1$. Note that (45) (in contrast to (34)) does not possess the property of chiral invariance. The transverse part of the vertex has a definite fixed form that does not contain arbitrary functions or coefficients. The presence of κ^2 in the denominator of the expression (47) indicates explicitly the non-perturbative nature of the solution.

An interesting solution also exists for a definite value of the bare quark mass, $m_0^2 = \kappa^2$ (it also corresponds to the case $Y(y) = \kappa y^{-1/2}$):

$$\begin{aligned} G(p) &= \frac{\not p}{p^2} , \\ \Gamma_\mu^{(\mathrm{L})}(p,q,q-p) &= \gamma_\mu , \\ \Gamma_\mu^{(\mathrm{T})}(p,q,q-p) &= \frac{3}{4\kappa^2} \frac{(\not p - \sigma\kappa)}{\sqrt{-p^2}} (\not p \gamma_\mu \not q - \not q \gamma_\mu \not p) \frac{(\not q - \sigma\kappa)}{\sqrt{-q^2}} , \end{aligned} \tag{48}$$

where $\sigma = m_0/\kappa = \pm 1$. As in the previous case, the transverse part of the vertex is fixed and has κ^2 in the denominator (non-perturbative nature). The expressions for the quark propagator and the longitudinal part of the vertex have the form of the free Green's functions for zero quark mass. In particular, the propagator (48) satisfies the Dirac equation for $m = 0$ (the solutions (31) and (45) do not possess this property).

4 Conclusions

In this paper, we have obtained a number of particular solutions of the Schwinger–Dyson equations for the lowest Green's functions with allowance for the corresponding gauge identities. Because of the arbitrariness in the choice of the vertex functions that occur in the equations, these solutions do not exhaust the set of all possible solutions, but their existence is instructive for the following reasons.

The result obtained in the second section, namely, that in the distinguished gauge (9) the free propagator and free vertex for the ghosts (7) agree with Eq. (11), is a further important argument for preferential use of this gauge in the infrared region of QCD. This conclusion, although it requires an additional investigation of the higher Schwinger–Dyson equations, suggests that the gauge $d = 4/(5-n)$ is distinguished by the fact that in it the ghosts do not at all distort (or do so only minimally) the gauge identities.

Study of the solutions for the quark propagator in the same distinguished gauge gives a new family of solutions that, generally speaking, depends functionally on the form of the transverse part of the vertex. When the condition (38) is imposed, one can obtain an explicit form of the solution, which depends on several parameters. Among the solutions found in this way are solutions with broken chiral invariance (for $m_0 = 0$). The obtaining of such explicitly non-perturbative solutions is of undoubted interest. We note that in applications of the scheme with the $1/N$ expansion solutions that break the chiral symmetry were obtained in [23]. A further investigation of the system (29)–(30) with a view to obtaining other classes of solutions would be interesting. The obtaining of a sufficiently large class of solutions would make it possible to consider the problem of the conditions imposed on the transverse part (on the function $f(p,q)$) by the higher Schwinger–Dyson equations.

On the other hand, the solutions obtained for the quark propagator and the quark-gluon vertex can be used to calculate the corresponding vacuum expectation values in QCD (condensates), as was done in [10, 24] for the solution (31)–(33), and in this sense compare them with experiment. It is possible that such a comparison in conjunction with future results from the investigation of their higher Schwinger–Dyson equations will make it possible to eliminate the existing arbitrariness and establish a definite solution.

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