

Triangle diagrams in the Standard Model

A. I. Davydychev and M. N. Dubinin

*Institute for Nuclear Physics, Moscow State University,
119899 Moscow, USSR*

Abstract

Method of massive loop Feynman diagrams evaluation developed in previous publications [3, 4, 5] is applied to the calculation of triangle diagrams appearing in the framework of the Standard Model. Representation of the final result in terms of hypergeometric functions is shown to give compact expressions, convenient for the following applications.

In: Proc. XIV Workshop "Problems on High Energy Physics and Field Theory"
(Protvino, July 1991), Nauka, Moscow, 1992, p. 294–299.

1. For the successful description of the experimental results measured on the new generation colliders we need the detailed information about loop radiative corrections to different processes of particle interactions. Special interest at present time is connected with the investigation of three-point one-loop (“triangle”) diagrams containing massive Standard Model particles (W and Z bosons, Higgs boson, heavy quarks).

Usually for the calculation of massive triangle diagrams one uses the results for scalar one-loop dimensionally-regularized three-point integrals presented in [1]. Integrals with the tensor structure in the numerator are reduced to scalar integrals with the help of the method given in [2]. Final expressions usually contain dilogarithms (Spence functions) of the complicated arguments.

In present paper we demonstrate an example of applying the new method of evaluating massive loop diagrams developed in [3, 4, 5]. This method is based on the representation of massive denominators in the form of contour Mellin–Barnes integrals. This technique makes possible to obtain results for arbitrary values of the space-time dimension and powers of the denominators. As a rule, final results are expressed in the form of hypergeometric functions (for the special values of the parameters they can be reduced to well-known expressions). We show that hypergeometric representation of the results is very convenient for analysis of behaviours in different regions of particle momenta and masses.

As an example of the application of this technique we consider the problem of radiative corrections calculation for the process of the Higgs decay into two quarks ($H \rightarrow q\bar{q}$). Let us note that if the mass of the Higgs particle is less than W^+W^- threshold, hadronic branching ratio is the most important. Decay width of $H \rightarrow q\bar{q}$ in the lowest order of perturbation theory has been obtained in [6],

$$\Gamma_0(H \rightarrow q\bar{q}) = \frac{N_c G_F m^2 M \beta_0^3}{4\sqrt{2} \pi}, \quad (1)$$

where m is the quark mass, $M \equiv M_H$ is the Higgs boson mass and $\beta_0 \equiv (1 - 4m^2/M^2)^{1/2}$.

The first calculation of the order α_s QCD corrections was done in [7]. It is important to note that a number of papers appeared recently (for example, [8, 9, 10]) with analogous calculations, but the results of [7] and [9] coincide with each other and contradict the result of [8]. At the same time, the result of [8] was applied for setting experimental restrictions on the Higgs mass at LEP 1 energies.

2. One-loop corrections to $H \rightarrow q\bar{q}$ process are presented by triangle diagram (two sides of the triangle are quark lines and the third side is gluon line), and by self-energy corrections to quark legs. See diagrams, for example, in [9]. We shall use the Feynman gauge. To handle infrared singularities we introduce as usually a small mass of the gluon λ ($\lambda \rightarrow 0$). We shall use the method given in [11, 12] for reduction of tensor integrals to scalar integrals rather than the standard technique [2].

We use the following notation for the scalar integrals:

$$J(n; \mu, \nu, \rho) \equiv \int \frac{d^n r}{(r^2 - \lambda^2)^\mu [(p - r)^2 - m^2]^\nu [(q - r)^2 - m^2]^\rho} \Big|_{\substack{p^2=q^2=m^2 \\ \lambda \rightarrow 0}}, \quad (2)$$

where $n = 4 - 2\varepsilon$ is the space-time dimension. Integrals (2) have the obvious symmetry with respect to two indices: $J(n; \mu, \nu, \rho) = J(n; \mu, \rho, \nu)$.

Let us evaluate the triangle diagram (we normalize it to the lowest order perturbation theory contribution). The expression for the triangle diagram in terms of scalar integrals (2) has the form

$$A = \frac{\alpha_s C_F}{4i\pi^3} \left\{ 2(2m^2 - k^2)J(n; 1, 1, 1) + (n - 4)J(n; 0, 1, 1) + 4J(n; 1, 1, 0) - 8m^2\pi^{-1}J(n + 2; 1, 2, 1) \right\}, \quad (3)$$

where C_F is the color factor, $k = p - q$ is the Higgs boson momentum, and the integral in $(n + 2)$ dimensions appeared after the application of the formula from [12].

It is easy to see that the infrared singularity (as $\lambda \rightarrow 0$) occurs only in $J(n; 1, 1, 1)$. Using the expressions obtained in [5] (with the notation $z = k^2/4m^2$) we get the following results for convergent and divergent parts of this integral ($n = 4$)

$$J(n; 1, 1, 1) = \frac{i\pi^2}{2m^2} \left\{ \ln \frac{\lambda^2}{m^2} {}_2F_1(1, 1; 3/2|z) + {}_{2(1)}F_1(1, 1; 3/2|z) \right\}. \quad (4)$$

Here ${}_2F_1$ is the Gauss hypergeometric function defined (at $|z| < 1$) by

$${}_2F_1(a, b; c|z) = \sum_{j=0}^{\infty} \frac{z^j}{j!} \frac{(a)_j (b)_j}{(c)_j}, \quad (5)$$

where $(a)_j \equiv \Gamma(a + j)/\Gamma(a)$ is the Pochhammer symbol, and

$${}_{2(1)}F_1(a, b; c|z) \equiv \frac{\partial}{\partial a} {}_2F_1(a, b; c|z) \quad (6)$$

is the derivative of ${}_2F_1$ with respect to the parameter a . From (5) and (6) we get (at $|z| < 1$)

$${}_{2(1)}F_1(a, b; c|z) = \sum_{j=0}^{\infty} \frac{z^j}{j!} \frac{(a)_j (b)_j}{(c)_j} [\psi(a + j) - \psi(a)], \quad (7)$$

where ψ function is the logarithmic derivative of Γ function (for example, $\psi(a + j) - \psi(a) = \sum_{l=0}^{j-1} (a + l)^{-1}$). In the following we shall need also the function

$${}_2F_{1(1)}(a, b; c|z) \equiv \frac{\partial}{\partial c} {}_2F_1(a, b; c|z) \quad (8)$$

defined at $|z| < 1$ by the series

$${}_2F_{1(1)}(a, b; c|z) = - \sum_{j=0}^{\infty} \frac{z^j}{j!} \frac{(a)_j (b)_j}{(c)_j} [\psi(c + j) - \psi(c)]. \quad (9)$$

Let us note that many useful properties of ${}_{2(1)}F_1$ and ${}_2F_{1(1)}$ functions (integral representations, analytical continuation properties, recursive formulas, formulas for special values of parameters and variables, etc.) can be obtained from the corresponding formulas for ${}_2F_1$ (see [13] for details).

The rest of integrals in expression (3) have no singularities in λ and it is very easy to obtain (as $\varepsilon \rightarrow 0$)

$$\begin{aligned}\pi^{-1}J(n+2; 1, 2, 1) &= i\pi^2 \left(-\frac{1}{2m^2}\right) {}_2F_1(1, 1; 3/2|z), \\ \varepsilon J(n; 0, 1, 1) &= i\pi^2, \quad J(n; 1, 1, 0) = i\pi^{2-\varepsilon}\Gamma(1+\varepsilon)\left\{\frac{1}{\varepsilon} - \ln \frac{m^2}{\mu^2} + 2\right\},\end{aligned}$$

where μ is the massive parameter of the dimensional regularization, and the $1/\varepsilon$ pole corresponds to the ultraviolet divergency that should be renormalized. Finally, the expression for the triangle diagram under consideration takes the form

$$\begin{aligned}A &= \frac{\alpha_s C_F}{2\pi^{1+\varepsilon}} \Gamma(1+\varepsilon) \left\{ \frac{1}{\varepsilon} - \ln \frac{m^2}{\mu^2} + 3 + \left[(1-2z) \ln \frac{\lambda^2}{m^2} - 2 \right] {}_2F_1(1, 1; 3/2|z) \right. \\ &\quad \left. + (1-2z) {}_{2(1)}F_1(1, 1; 3/2|z) \right\}.\end{aligned}\quad (10)$$

3. In order to obtain the standard representation for Euclidean momentum of the Higgs particle ($z = k^2/4m^2 < 0$), we use the formulas

$${}_2F_1(1, 1; 3/2|z) = \frac{\beta^2 - 1}{2\beta} \ln \frac{\beta + 1}{\beta - 1}, \quad (11)$$

$${}_{2(1)}F_1(1, 1; 3/2|z) = \frac{\beta^2 - 1}{\beta} \left\{ \text{Li}_2 \left(-\frac{\beta - 1}{\beta + 1} \right) + \ln \frac{\beta + 1}{2\beta} \ln \frac{\beta + 1}{\beta - 1} - \frac{1}{4} \ln^2 \frac{\beta - 1}{\beta + 1} + \frac{\pi^2}{12} \right\}, \quad (12)$$

where $\beta = (1 - 4m^2/k^2)^{1/2} = (1 - z^{-1})^{1/2}$, and Li_2 is Euler dilogarithm. These formulas can be obtained from the parametrical integral representations for ${}_2F_1$ and ${}_{2(1)}F_1$ (see [13]). Inserting (11) and (12) into (10) we come to the same result as in [9].

In order to obtain the threshold behaviour at $k^2 \simeq 4m^2$ ($z \gtrsim 1$), it is convenient to use the analytic continuation formulas from the variable z to $(1-z)$ for the functions ${}_2F_1$ and ${}_{2(1)}F_1$ (see [13]). In our case they take the form

$${}_2F_1(1, 1; 3/2|z) = \frac{\pi}{2} \frac{1}{\sqrt{z(1-z)}} - {}_2F_1(1, 1; 3/2|1-z), \quad (13)$$

$$\begin{aligned}{}_{2(1)}F_1(1, 1; 3/2|z) &= -\frac{\pi}{2} \frac{\ln[4(1-z)]}{\sqrt{z(1-z)}} + 2 {}_2F_1(1, 1; 3/2|1-z) \\ &\quad - {}_{2(1)}F_1(1, 1; 3/2|1-z) - {}_2F_{1(1)}(1, 1; 3/2|1-z),\end{aligned}\quad (14)$$

where ${}_2F_{1(1)}$ is defined by (8)–(9). Using (13)–(14) we have for $z > 1$ the following results:

$$\text{Re } {}_2F_1(1, 1; 3/2|z) = \frac{\beta^2 - 1}{2\beta} \ln \frac{1 + \beta}{1 - \beta}, \quad (15)$$

$$\text{Re } {}_{2(1)}F_1(1, 1; 3/2|z) = \frac{\beta^2 - 1}{\beta} \left\{ \text{Li}_2 \left(\frac{1 - \beta}{1 + \beta} \right) + \ln \frac{1 + \beta}{2\beta} \ln \frac{1 + \beta}{1 - \beta} - \frac{1}{4} \ln^2 \frac{1 + \beta}{1 - \beta} + \frac{\pi^2}{3} \right\}. \quad (16)$$

If $M_H^2 > 4m^2$ we can set $k^2 = M_H^2$ ($\beta = \beta_0$). Substitution of (15) and (16) into (10) also gives the same result as in [9].

Besides this we calculated other contributions of the order α_s (soft and hard gluons emission) and verified the results of [9]. In the final result infrared and collinear singularities cancel.

4. In present paper we considered the example of applying the method presented in [3, 4, 5] to triangle diagrams in the Standard Model and confirmed the result of [9]. For the loop QCD correction of order α_s to the process $H \rightarrow q\bar{q}$ we obtained the representation (10) in terms of hypergeometric functions. From our point of view, this representation is more convenient for the analysis of diagrams behaviour in different regions. Expressions (5), (7) and (9) represent the series with known coefficients. Analytic continuation formulas (13) and (14) make possible to investigate threshold behaviour. All threshold singularities (square root and logarithmic) are factorized explicitly, and the rest hypergeometric functions represent regular (near the threshold) expansions with known coefficients. In the same way it is possible to investigate the case $z \rightarrow \infty$. The correspondence formulas (11)–(12) and (15)–(16) enable us to pass from the hypergeometric representation to standard expressions in terms of dilogarithms of complicated arguments. This obstacle could be useful in numerical calculations.

In the nearest future we are planning to consider the application of method [3, 4, 5] to some more complicated loop diagrams.

The authors are grateful to E. E. Boos and K. Hikasa for useful discussions.

References

1. G. 'tHooft, M. Veltman // Nucl. Phys. 1979. Vol. B153. P. 365.
2. G. Passarino, M. Veltman // Nucl. Phys. 1979. Vol. B160. P. 151.
3. E. E. Boos, A. I. Davydychev // Teor. Mat. Fiz. 1991. Vol. 89. P. 56.
4. A. I. Davydychev // J. Math. Phys. 1991. Vol. 32. P. 1052.
5. A. I. Davydychev // J. Math. Phys. 1992. Vol. 33. P. 358.
6. L. Resnick, M. Sundaresan, P. Watson // Phys. Rev. 1973. Vol. D8. P. 172.
7. E. Braaten, J. Leveille // Phys. Rev. 1980. Vol. D22. P. 715.
8. P. Janot // Phys. Lett. 1989. Vol. B223. P. 110.
9. M. Drees, K. Hikasa // Phys. Lett. 1990. Vol. B240. P. 455.
10. D. Yu. Bardin, B. M. Vilensky, P. Kh. Khristova // Yad. Fiz. 1991. Vol. 53. P. 240.
11. B. A. Arbuzov, E. E. Boos, S. S. Kurennoy, K. Sh. Turashvili // Yad. Fiz. 1984. Vol. 40. P. 836.
12. A. I. Davydychev // Phys. Lett. 1991. Vol. B263. P. 107.
13. A. Erdelyi, W. Magnus, F. Oberhettinger, F. G. Tricomi // Higher transcendental functions (Bateman manuscript project), Vol. 1 - New York: McGraw-Hill. 1953.