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MOMENTUM EXPANSION OF TWO-LOOP SELF-ENERGY DIAGRAMS OCCURRING IN THE STANDARD MODEL

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We consider an algorithm for constructing the power series (in the external momentum) of two-loop two-point diagrams with arbitrary masses of the internal particles. Comparison with a numerical calculation of some two-loop diagrams occurring in the Standard Model shows that the first few terms of the expansion provide good approximations in the region when the external momentum is below the threshold.

1. Many interesting problems of the Standard Model (and its extensions) are connected with calculation of Feynman loop diagrams with massive particles. For example, we should keep the masses of W and Z bosons, heavy quarks, Higgs particles, etc. Numerous applications are connected with the evaluation of radiative corrections to cross sections and decay widths, etc.

Some special cases of massive two-loop propagator-type diagrams (like QED corrections to the photon self-energy) have been considered in several publications (see, e.g., [1, 2]). On the other hand, the problem of evaluating such diagrams when all the internal particles are massive is more complicated, and exact expressions are not known. For example, in ref. [3] the result was presented in terms of a two-fold integral representation.

In the present paper we consider an algorithm to obtain the expansion of two-loop massive self-energy diagrams in the external momentum, the coefficients of the expansion being calculated analytically. We construct the expansion for the general case of two-loop self-energy integrals (with five different masses). The coefficients are represented in terms of vacuum two-loop integrals with different powers of denominators, each of them depending on no more than three different masses only. We present some results for these vacuum integrals, and we also construct recurrence relations for evaluating these integrals with higher powers of denominators. We apply the considered algorithm to some diagrams contributing to the Z boson and photon self-energies in the Standard Model and compare our expansions with the results obtained by a numerical program based on the integral representation [3].

2. For integer values of the powers of denominators ν_i, all possible two-loop self-energy contributions can be reduced to scalar integrals corresponding to the diagram

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presented in Fig. 1. The corresponding Feynman integral is of the following form:

$$J(\nu_1,...,\nu_5;m_1,...,m_5;k)$$

$$\equiv \iint \frac{d^n p \;\; d^n q}{((k-p)^2-m_1^2)^{\nu_1}\,((k-q)^2-m_2^2)^{\nu_2}\,((p-q)^2-m_3^2)^{\nu_3}\,(p^2-m_4^2)^{\nu_4}\,(q^2-m_5^2)^{\nu_5}}, \tag{1}$$

where k is the external momentum (note that J really depends on k^2) and $n = 4-2\varepsilon$ is the space-time dimension (in the framework of dimensional regularization [4]). Here and below the usual causal way of dealing with denominators in pseudo-Euclidean momentum space $((k^2)^{-\nu} \leftrightarrow (k^2+i0)^{-\nu})$ is understood. More simple two-loop diagrams (with four or three internal lines), as well as products of one-loop diagrams, can be obtained from (1) by putting some of the powers of denominators ν_i equal to zero. Moreover, by using (an appropriate number of times) the obvious decomposition formula

$$\frac{1}{(p^2-m^2)(p^2-m'^2)} = \frac{1}{m^2-m'^2}\left(\frac{1}{p^2-m^2} - \frac{1}{p^2-m'^2}\right), \tag{2}$$

the two-loop diagram with another “topology” also can be reduced to integrals (1) (this is true if the powers of denominators are integer). So, it is sufficient to consider the scalar integrals (1) with different powers of denominators.

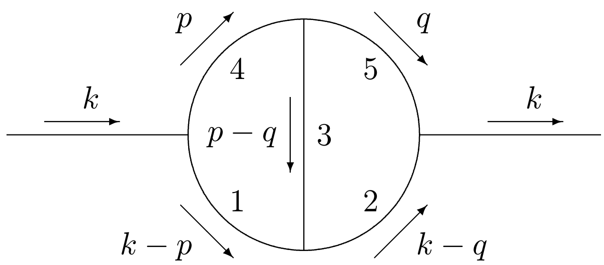

Figure 1:

In the massive case the integrals (1) are known to be regular functions of k^2 as $k^2 \to 0$. In what follows we shall examine the momentum power series expansion of these integrals. By using the momentum space d'Alembertian,

$$\Box_k \equiv \frac{\partial^2}{\partial k_\mu \partial k^\mu} \quad , \tag{3}$$

the following Taylor-type expansion can be derived for a regular (at $k^2=0$) scalar function :

$$\begin{aligned} J(k^2) \quad &= \quad J(0) + \frac{k^2}{2n}\left(\Box_k J(k^2)\right)|_{k=0} + \frac{(k^2)^2}{8n(n+2)}\left(\Box_k^2 J(k^2)\right)|_{k=0} + ... \\ &= \quad \sum_{j=0}^{\infty} \frac{1}{j!\;(n/2)_j}\left(\frac{k^2}{4}\right)^j \left(\Box_k^j J(k^2)\right)|_{k=0}\,, \end{aligned} \tag{4}$$

where $(a)_j \equiv \Gamma(a+j)/\Gamma(a)$ is the Pochhammer symbol. Note that for the special case $m_1 = m_2 = m_4 = m_5,\;\; m_3 = 0$ an analogous expansion has been considered, e.g., in ref. [5] (see also [6] and references therein).

The result of applying the operator (3) to the integral (1) is (we only write shifted arguments of the integrals on the r.h.s.):

$$\Box_k J(\nu_1,...,\nu_5;m_1,...,m_5;k)$$
$$= 4\left\{(\nu_1+\nu_2+1-n/2)\left(\nu_1\ J(\nu_1+1)+\nu_2\ J(\nu_2+1)\right)\right.$$
$$+\nu_1(\nu_1+1)m_1^2\ J(\nu_1+2)+\nu_2(\nu_2+1)m_2^2\ J(\nu_2+2)$$
$$\left.+\nu_1\nu_2\left((m_1^2+m_2^2-m_3^2)\ J(\nu_1+1,\nu_2+1)-J(\nu_1+1,\nu_2+1,\nu_3-1)\right)\right\}. \quad (5)$$

When we need to evaluate $\Box_k^2 J$ we apply the second $\Box_k$ to the r.h.s. of (5), and so on.

After applying $\Box_k^j$ we put $k=0$. As a result, we obtain vacuum integrals (without external momentum). By using, when necessary, the decomposition (2), these vacuum integrals can be reduced to integrals with three denominators,

$$I(\nu_1,\nu_2,\nu_3;m_1,m_2,m_3) \equiv \int\int \frac{d^n p\ d^n q}{(p^2-m_1^2)^{\nu_1}\,(q^2-m_2^2)^{\nu_2}\,((p-q)^2-m_3^2)^{\nu_3}}, \quad (6)$$

(see Fig. 2). Note that in these integrals we may have m_4 instead of m_1 or m_5 instead of m_2. So, the coefficients of the momentum expansion (4) of the integrals

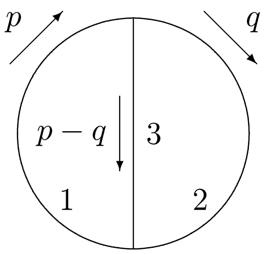

Figure 2:

(1) can be expressed in terms of vacuum integrals (6) with various (integer) values of ν_1,ν_2,ν_3.

When one of the indices ν_i is zero the result corresponds to the product of two massive tadpoles and can be represented in terms of gamma functions. For example:

$$I(\nu_1,\nu_2,0) = \pi^n i^{2-2n}(-m_1^2)^{n/2-\nu_1}(-m_2^2)^{n/2-\nu_2}\frac{\Gamma(\nu_1-n/2)\Gamma(\nu_2-n/2)}{\Gamma(\nu_1)\Gamma(\nu_2)}. \quad (7)$$

Here and below it is understood that $i^{-n}(-m^2)^{n/2}=(m^2)^{n/2}$.

Note that when we apply operator (3) to (1) we may also obtain negative values of ν_3 (the corresponding denominator “transforms” into numerator). Such contributions can also be reduced to products of two massive tadpoles.

So, the main problem in obtaining the coefficients of the expansion of (1) in k^2 is to evaluate the integrals (6) for positive integer values of ν_1,ν_2 and ν_3.

3. In this section we present results for the vacuum integrals (6) obtained in the paper [7]. Note that some of the results of such type have been obtained also in refs. [8, 9, 10, 11] (we checked the correspondence of results whenever possible).

We used the method of evaluating massive Feynman integrals [12, 13] based on the Mellin–Barnes representation of massive denominators. In such a way, it is possible to obtain the result for the general case of (6) with arbitrary ν_i and m_i (see in [7]). This expression contains hypergeometric functions of two variables.

If we consider the case $\nu_1 = \nu_2 = \nu_3 = 1, n \to 4$ $(\varepsilon \to 0)$, the result reduces to

$$I(1,1,1;m_1,m_2,m_3) = \pi^{4-2\varepsilon}(m_3^2)^{1-2\varepsilon}\frac{A(\varepsilon)}{2}$$
$$\times \left\{ -\frac{1}{\varepsilon^2}(1+x+y) + \frac{2}{\varepsilon}(x\ln x + y \ln y) \right.$$
$$\left. -x\ln^2 x - y\ln^2 y + (1-x-y)\ln x \ln y - \lambda^2 \Phi(x,y) \right\}, \tag{8}$$

where

$$x \equiv \frac{m_1^2}{m_3^2} \quad , \quad y \equiv \frac{m_2^2}{m_3^2} \ , \tag{9}$$
$$\lambda(x,y) \equiv \sqrt{(1-x-y)^2 - 4xy} \ , \tag{10}$$

and

$$A(\varepsilon) \equiv \frac{\Gamma^2(1+\varepsilon)}{(1-\varepsilon)(1-2\varepsilon)}$$
$$= 1 + \varepsilon(3-2\gamma) + \varepsilon^2\left(7 - 6\gamma + 2\gamma^2 + \frac{\pi^2}{6}\right) + O(\varepsilon^3), \tag{11}$$

($\gamma = 0.57721566...$ is Euler's constant). The function $\Phi(x,y)$ in (8) has the following representations:

$$\Phi(x,y) = \frac{1}{\lambda}\left\{ 2\ln\left(\frac{1+x-y-\lambda}{2}\right)\ln\left(\frac{1-x+y-\lambda}{2}\right) - \ln x \ln y \right.$$
$$\left. -2\,\mathrm{Li}_2\left(\frac{1+x-y-\lambda}{2}\right) - 2\,\mathrm{Li}_2\left(\frac{1-x+y-\lambda}{2}\right) + \frac{\pi^2}{3} \right\} \tag{12}$$

if $\sqrt{x} + \sqrt{y} \le 1$ $(\lambda^2 \ge 0)$, and

$$\Phi(x,y) = \frac{2}{\sqrt{-\lambda^2}}\left\{ \mathrm{Cl}_2\left(2\arccos\left(\frac{-1+x+y}{2\sqrt{xy}}\right)\right) \right.$$
$$\left. +\mathrm{Cl}_2\left(2\arccos\left(\frac{1+x-y}{2\sqrt{x}}\right)\right) + \mathrm{Cl}_2\left(2\arccos\left(\frac{1-x+y}{2\sqrt{y}}\right)\right) \right\}. \tag{13}$$

if $\sqrt{x} + \sqrt{y} \ge 1$ $(\lambda^2 \le 0)$. Here Li_2 denotes dilogarithm,

$$\mathrm{Li}_2(\xi) = -\int_0^1 dt \frac{\ln(1-\xi t)}{t} \ , \tag{14}$$

and Cl_2 corresponds to the Clausen's integral function,

$$\mathrm{Cl}_2(\theta) = -\int_0^\theta d\theta \ \ln\left|2\sin\frac{\theta}{2}\right| . \tag{15}$$

(for details see in [14]).

Note that by permutation of m_1, m_2, m_3 (in the representation (8), (12)) we can also obtain results for the region $\sqrt{x} - \sqrt{y} \geq 1$ and the region $\sqrt{y} - \sqrt{x} \geq 1$. It is interesting to note that the function $\Phi(x, y)$ (12) is the same as in the case of the massless triangle diagram (see, e.g., [15]) where x and y are constructed from external momenta squared instead of masses. It should be also noted that the result (8), (13) is completely symmetric in m_1, m_2, m_3.

If two of the masses are equal (for example, $m_1 = m_2 \equiv m,\ m_3 \equiv M$), it is convenient to use the following notations:

$$z \equiv \frac{M^2}{4m^2} \quad , \quad \lambda(z) = \sqrt{1 - \frac{1}{z}} \quad , \quad \Phi(z) \equiv \Phi\left(\frac{1}{4z}, \frac{1}{4z}\right). \tag{16}$$

The corresponding result is

$$I(1,1,1; m, m, M) = \pi^{4-2\varepsilon} (m^2)^{1-2\varepsilon}\ A(\varepsilon)$$
$$\times \left\{ -\frac{1}{\varepsilon^2}(1+2z) + \frac{1}{\varepsilon}(4z \ln(4z)) - 2z \ln^2(4z) + 2(1-z)\Phi(z) \right\} \tag{17}$$

where

$$\Phi(z) = \frac{1}{\lambda}\left[-4\,\mathrm{Li}_2\left(\frac{1-\lambda}{2}\right) + 2\ln^2\left(\frac{1-\lambda}{2}\right) - \ln^2(4z) + \frac{\pi^2}{3} \right] \tag{18}$$

if $z \geq 1$, and

$$\Phi(z) = 4\sqrt{\frac{z}{1-z}}\ \ \mathrm{Cl}_2\left(2 \arcsin\sqrt{z}\right) \tag{19}$$

if $z \leq 1$. In particular, if $M = m$ $(z = 1/4)$ we have

$$I(1,1,1; m, m, m) = \pi^{4-2\varepsilon} (m^2)^{1-2\varepsilon} A(\varepsilon) \left\{ -\frac{3}{2\varepsilon^2} + \frac{27}{2} S_2 \right\} \tag{20}$$

with (we follow the notation of ref. [2])

$$S_2 = \frac{4}{9\sqrt{3}} \mathrm{Cl}_2\left(\frac{\pi}{3}\right) = 0.2604341... \tag{21}$$

where $\mathrm{Cl}_2\,(\pi/3) = 1.0149417...$ corresponds to the maximum of Clausen's integral [14] and cannot be represented in terms of other known transcendental constants. This constant has appeared before in two-loop massive calculations (see, e.g., [8, 2, 10]).

4. When we calculate the coefficients of the expansion (4) we also need to evaluate integrals (6) with higher powers of denominators. To obtain results for these integrals, it is convenient to use the integration-by-parts technique [16]; for example:

$$\int\int d^n p\ d^n q \frac{\partial}{\partial p_\mu} \left\{ \frac{p_\mu}{(p^2 - m_1^2)^{\nu_1}\,(q^2 - m_2^2)^{\nu_2}\,((p+q)^2 - m_3^2)^{\nu_3}} \right\} = 0. \tag{22}$$

Identities of the type of (22) make it possible to construct a recursive procedure for evaluating $I(\nu_1, \nu_2, \nu_3)$ (in this section we omit the arguments m_1, m_2, m_3) with integer ν's (by analogy with [15]).

For the integrals (6), we get three independent conditions. The determinant of the corresponding system of equations for $I(\nu_1+1,\nu_2,\nu_3)$, $I(\nu_1,\nu_2+1,\nu_3)$ and $I(\nu_1,\nu_2,\nu_3+1)$ is

$$\begin{aligned}\Delta(m_1^2,m_2^2,m_3^2) &= 2(m_1^2m_2^2+m_1^2m_3^2+m_2^2m_3^2)-(m_1^4+m_2^4+m_3^4) \\ &= -m_3^4\lambda^2(x,y)\end{aligned} \tag{23}$$

where $\lambda(x,y)$ is defined by (10). Solving these equations yields

$$\begin{aligned} I(\nu_1+1,\nu_2,\nu_3) = {} & \frac{1}{\nu_1 m_1^2\Delta(m_1^2,m_2^2,m_3^2)} \\ & \times\left\{\left[\nu_2(m_1^2-m_3^2)(m_1^2-m_2^2+m_3^2)+\nu_3(m_1^2-m_2^2)(m_1^2+m_2^2-m_3^2)\right.\right. \\ & \left.+nm_1^2(-m_1^2+m_2^2+m_3^2)-\nu_1\Delta(m_1^2,m_2^2,m_3^2)\right]I(\nu_1,\nu_2,\nu_3) \\ & +\nu_2m_2^2(m_1^2-m_2^2+m_3^2)\left[I(\nu_1,\nu_2+1,\nu_3-1)-I(\nu_1-1,\nu_2+1,\nu_3)\right] \\ & \left.+\nu_3m_3^2(m_1^2+m_2^2-m_3^2)\left[I(\nu_1,\nu_2-1,\nu_3+1)-I(\nu_1-1,\nu_2,\nu_3+1)\right]\right\} \end{aligned} \tag{24}$$

and expressions for $I(\nu_1,\nu_2+1,\nu_3)$ and $I(\nu_1,\nu_2,\nu_3+1)$ (which can be obtained from (24) by permutation of indices). These results make it possible to evaluate integrals with $\nu_1+\nu_2+\nu_3=\sigma+1$ in terms of the integrals with $\nu_1+\nu_2+\nu_3=\sigma$.

Formula (24) (and permutations) enable one to evaluate integrals with any positive integer ν's in terms of $I(1,1,1)$ and trivial boundary integrals (7) (when one of ν's is equal to zero). These recurrence relations can easily be algorithmized (to do this, we have used the REDUCE system [17]). As an example, it is easy to check that the result for $I(2,1,1)$ (obtained by applying relation (24) in the case $\nu_1=\nu_2=\nu_3=1$) after some transformations coincides with the results presented in refs. [8, 9] in terms of dilogarithms.

In the case $m_1=m_2\equiv m,\;\; m_3\equiv M$ $(z\equiv M^2/(4m^2))$ we get (at $\varepsilon=0$)

$$I(2,2,1)=\frac{\pi^4}{2m^2}\frac{1}{(1-z)}\left\{(1-2z)\frac{\mathrm{Cl}_2\left(2\arcsin\sqrt{z}\right)}{\sqrt{z(1-z)}}+\ln(4z)\right\},\quad z\le 1, \tag{25}$$

$$\begin{aligned} I(2,2,1) = {} & \frac{\pi^4}{2m^2}\frac{1}{(1-z)}\left\{\ln(4z)\right. \\ & \left.+\frac{2z-1}{\lambda z}\left[\mathrm{Li}_2\left(\frac{1-\lambda}{2}\right)-\frac{1}{2}\ln^2\left(\frac{1-\lambda}{2}\right)+\frac{1}{4}\ln^2(4z)-\frac{\pi^2}{12}\right]\right\},\quad z\ge 1. \end{aligned} \tag{26}$$

These results give the first term of expansion (4) for the integral corresponding to the diagram in Fig. 1 with $m_1=m_2=m_4=m_5=m,\; m_3=M$. Note that expressions (25) and (26) coincide at $z=1$ and yield

$$I(2,2,1)=\frac{\pi^4}{3m^2}(4\ln 2-1),\qquad z=1\qquad (M=2m)\,. \tag{27}$$

5. We shall demonstrate our algorithm by considering the integral

$$J(m_1,m_2,m_3,m_4,m_5;k)\equiv J(1,1,1,1,1;m_1,m_2,m_3,m_4,m_5;k). \tag{28}$$

It is finite and it can be calculated for arbitrary masses by a method described in ref. [3] which involves a two-dimensional numerical integration. We shall approximate the integral (28) by taking the first few terms of its momentum expansion, and then compare the results with the values we obtain by using the method in ref. [3].

In our first example we take $m_1 = m_2 = m_4 = m_5 = m$ and $m_3 = M$ and write the momentum expansion as:

$$J(m,m,M,m,m;k) = \frac{\pi^4}{m^2} \sum_{j=0}^{\infty} c_j(z) \left(\frac{k^2}{4m^2} \right)^j , \tag{29}$$

with $z = M^2/(4m^2)$. By performing the procedure described in the previous sections we obtain expressions for the coefficients $c_j(z)$ that contain z, $\ln z$ and $\Phi(z)$. In the denominator they contain powers of z and $(1-z)$, that are introduced when the recurrence relation (24) is used. To avoid numerical instabilities, we derive expansions of our expressions in z and in $(z-1)$ which we use when z is close to 0 or 1. Near $z = 1$ the coefficients $c_j(z)$ are analytic. The expansions near $z = 0$ contain powers of z, and in addition terms of the form $z^l \ln z$.

Fig. 3 shows the first terms of the momentum expansion (29), and also the values of J obtained by numerical integration (these values are indicated by crosses), in the region below the threshold at $k^2 = 4m^2$ (the right edge of the plot). In this example we chose $M = M_Z = 91.16$ GeV and $m = m_t = 140$ GeV, the masses of the Z boson and the top quark. For the latter we picked a value which is in the middle of the allowed range of masses. This combination of masses occurs in two-loop corrections to the Z and photon propagators in the Standard Model. The straight line (in Fig. 3) is the first term of the momentum expansion (corresponding to $c_0(z)$. The curves show the improvement of the approximation when the higher terms of the momentum expansion (corresponding to $c_1(z), ..., c_5(z)$) are added. So, the last (upper) curve contains six terms of the expansion (29). In this particular case only three terms are needed to obtain an accuracy of 1% when $k^2/m_t^2 = 1$. To obtain 1% accuracy when $k^2/m_t^2 = 2$, five terms are needed. Notice that if $k^2 \approx M_Z^2$, $k^2/m_t^2 \approx 0.42$, which is in the region where the momentum expansion approximation works well.

An example involving three different masses is shown in Fig. 4, where $m_1 = m_4 = m_t = 140$ GeV, $m_2 = m_5 = M_W = 80$ GeV (the W boson mass) and $m_3 = m_b = 5$ GeV (the b quark mass). In this case the first threshold is at $k^2 = 4M_W^2$. If we choose $k^2 \approx M_Z^2$, $k^2/M_W^2 \approx 1.3$, so we are again well below the threshold.

6. Thus, in the present paper we considered an algorithm for constructing the expansion in the momentum squared for the general case of massive two-loop propagator-type diagrams. We showed that the coefficients of the expansion can be expressed in terms of vacuum two-loop integrals, each of them involving no more than three different masses. For all integer powers of denominators, these integrals can be evaluated in terms of the function $\Phi(x,y)$ involving dilogarithms (12) or Clausen functions (13), and powers and logarithms of the masses. In such a way, we obtain analytical results for coefficients of the expansion. It should be noted that, because the main formulae of the algorithm are valid for arbitrary n, they can be applied also to ultraviolet divergent integrals that occur in realistic calculations.

We considered some physically interesting cases (the diagrams contributing to the Z boson and photon self-energies), and we checked that our algorithm provides a rapidly converging series in the region below the first threshold. Moreover, the presented algorithm can also be generalized to calculate massive three-point two-loop diagrams, if all the external momenta squared lie below the corresponding thresholds. In other cases, when one is interested in the region near or beyond the threshold, the expansion for small momenta cannot be used, and one needs a modified procedure. For example, for the large momentum behaviour (above the thresholds), a general technique of asymptotic expansions can be used (see, e.g., [6] and references therein).

It should be noted that momentum expansions with analytically calculated coefficients could be especially useful (as compared with the numerical integration) in cases when one deals with particles whose masses are not fixed (e.g. the Higgs boson and t quark masses in the Standard Model).

Acknowledgements. We are grateful to F.A. Berends and W.L. van Neerven for their assistance and helpful discussions. We are also grateful to D.J. Broadhurst, D. Kreimer, R. Scharf and V.A. Smirnov for some useful discussions, to H. Kuijf and S.C. van der Marck for writing the program that calculates the integral (28) numerically. One of the authors (A.D.) would like to thank the Instituut-Lorentz for their hospitality and the Nederlandse organisatie voor wetenschappelijk onderzoek (NWO) for financial support.

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