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Infrared problems of desribing the glueball and an estimation of its mass

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Abstract. The Bethe–Salpeter equation for the wave function of the bound state of two gluons is considered, and the expected region of momenta is discussed. In this connection we consider some issues concerning the form of the gluon propagator in infrared and ultraviolet regions and the possible influence of ghost Green's functions. The contributions to both sides of the Bethe–Salpeter equation are separated in "soft" and "hard" parts, in the spirit of the QCD sum rules method. In the leading approximation of the presented approach the value $M_{\rm gl} = 1.25$ GeV for the 0^{++} glueball mass is obtained, and the masses of 0^{++} and 2^{++} glueballs appear to be degenerate. These results are in a good agreement with other approaches.

1. Introduction

The glueball ("nuclear glue" particle) is an object that has been treated by theorists and experimenters for about 15 years. Although the idea that colorless gluonic bound states should exist in QCD [1] looks very natural, the glueball problem as a whole appeared to be extremely complicated. As regards the experimental aspects, the problem gets complicated because the mere discovery of a new resonance with appropriate quantum numbers is far from being sufficient and makes it necessary to prove also that some of the properties of this resonance are unusual in terms of its interpretation as a quark state, and that the given properties can be understood if the found resonance is assumed to be a gluonic state [2]. The problem is actually even more complicated because of possible mixing. Several candidates to glueballs with different quantum numbers have been found experimentally by now, for example, the states with $J^{PC} = 0^{++}$ — $f_0(1240)$ [3] and $G(1590)$ [4], 2^{++} — $\theta(1720)$ [5], 0^{-+} — $\iota(1440)$ [6], etc. In particular, as shown in [4], the unusual properties of $G(1590)$ consist in that the probability of its decay into π- and K-mesons is 3–5 times lower compared with its decay into the $\eta\eta$-pair (and still lower than that into the $\eta\eta'$-pair). This circumstance permits G to be regarded as a possible candidate to glueballs, although the interpretation of G as the hybrid $gq\bar{q}$-state cannot be excluded [7].

The theoretical difficulties in describing the gluonic states arise mainly because of the necessity to consider the long distances where the perturbation theory is inapplicable. This means that various non-perturbative methods have to be used. At present, quite a number of different non-perturbative approaches to calculating the glueball state characteristics are known. Namely,
(i) Monte-Carlo lattice calculations (see, e.g., some of the recent works [8, 9, 10, 11]),
(ii) calculations by the QCD sum rules method [12],
(iii) estimates in the framework of the bag model [13],
(iv) estimates in terms of some of the versions of potential models [14],
(v) application of the method of effective Lagrangians and solving the corresponding Dyson-type equations [15],
(vi) estimates by the Rayleigh–Ritz method [16].

All the approaches predict the glueball masses to range from 0.8 to 2 and more GeV. However, the calculations by each of the above methods are not free of drawbacks and of some not quite justified assumptions. A serious review must be devoted to detailed examination of the above-mentioned methods and of the results obtained (to an extent, the recent review [17] meets this requirement). All the above demonstrates that the problem of searching for and describing the glueballs has been by no means solved and needs being studied further.

Let us consider, first of all, in what way we can very naively imagine that two (or more) massless particles (gluons) form a sufficiently heavy bound state. Obviously, if the gluon interaction is of negative sign (in fact, the very concept of potential is not quite correct in the case of massless particle interaction; the reasoning in terms of quasi-potential and Bethe–Salpeter kernel is more suitable), the binding energy is negative, and two massless particles will never form a bound state. If, however, the potential rises and is positive, such a bound state is quite possible to occur. If, by analogy with quarks, we assume that a linearly-rising "string" potential $V \sim a^2 r$ ($a \simeq 420$ MeV) is effective among gluons, we shall obtain the following qualitative estimate of the region of the characteristic relative momenta of glueball (with a mass of, e.g., 1.5 GeV):

$$M_{\rm gl} \simeq a^2 r_{\rm rel} \; ; \qquad q^2_{\rm rel} \sim \frac{1}{r^2_{\rm rel}} \simeq \frac{a^4}{M^2_{\rm gl}} \simeq 1.4 \cdot 10^{-2}\ {\rm GeV}^2 \; .$$

So, from the naive reasoning presented above we may expect that the characteristic momentum region for a glueball is that where the total momentum $P^2 = M^2_{\rm gl}$ is much greater than the relative momentum $q^2_{\rm rel}$. This qualitative circumstance will be used below as a hint when examining the Bethe–Salpeter equation for the wave function of the bound state of two gluons [18].

The idea of the present work is to extract the leading contribution to the kernel of the Bethe–Salpeter equation in the discussed range of momenta, using certain information about the infrared behaviour of gluon propagator, and to separate the contributions to both parts of the equation into "soft" and "hard" ones in the spirit of QCD sum rules method [12].

The structure of the paper is as follows. Section 2 presents the information about the infrared behaviour of the gluon Green's functions to be used henceforth and discusses the conditions under which the contributions of the ghost Green's functions appear to be insignificant. Section 3 examines the Bethe–Salpeter equation for the glueball wave function and gives an estimate of the glueball mass. Section 4 is a brief discussion of the results obtained and the accuracy of the estimate and presents a comparison with the results obtained in terms of other approaches.

2. The gluon and ghost Green's functions

It is well known that the phenomenon of dimensional transmutation takes place in QCD, and the scale $\Lambda_{\rm QCD} \sim 100 - 200$ MeV arises which prescribes the evolution of the running coupling constant $\alpha_{\rm s}(q^2)$. In this case, in the range of virtualities q^2 exceeding a certain characteristic momentum $k_0^2 \sim 3 - 5\ \Lambda^2_{\rm QCD}$, the $\alpha_{\rm s}$ value is small in virtue of asymptotic freedom, so the perturbation theory is quite applicable within this range (in the QCD sum rules method the contributions of high virtualities are associated with the coefficient functions [12]). In the range of small $q^2 \ll k_0^2$ the perturbation theory is inapplicable, the non-perturbative effects get operative and, for example, in the sum rules method the corresponding "soft" contributions are parametrized by phenomenological numbers, condensates. The estimate $k_0 \sim 700$ Mev was obtained in [19] when calculating the gluon condensates of lowest dimensions.

Studying the Bethe–Salpeter equation for the wave function of bound state of two gluons (glueball) requires information about the behaviour of the gluon Green's functions in different momenta regions (here and henceforth we use the Euclidean momentum space). We shall be particularly interested in the infrared and ultraviolet behaviour of the gluon propagator $D^{ab}_{\mu\nu}(k) = \delta^{ab} D_{\mu\nu}(k)$ which may be presented in the general case as

$$D_{\mu\nu}(k) = \frac{Z(k^2)}{k^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + f(k^2) \frac{k_\mu k_\nu}{k^2} \; , \tag{1}$$

where $f(k^2)$ corresponds to gauge-fixing term (see, e.g., in [20]; the choice of this function will be discussed below). As to the function $Z(k^2)$, it was noted above that this function

was quite properly described by the perturbation theory in the ultraviolet region and that $Z(k^2) = 1$ in the lowest order. In the infrared region (at $k^2 \ll k_0^2$) there are quite a number of serious arguments for the behaviour $Z(k^2) \sim M^2/k^2$, namely,
(i) the results of studying the Schwinger–Dyson equation for the gluon propagator in the infrared region taking account of the gauge identities [21, 22, 23] (see also the review [24]);
(ii) a good agreement of the values of the lowest-dimension condensates with the corresponding values obtained by the QCD sum rules [19, 25];
(iii) such an asymptotics corresponds to the linearly-rising "string" potential [26]; this agrees with the Monte-Carlo lattice calculations [27], with the analysis of heavy-quarkonium spectra [28] and with the results in the lattice QCD concerning the asymptotic behaviour of the β-function, $\beta(g) \to -g$ as $g \to \infty$ [29];
(iv) the possibility of understanding the chiral invariance breaking and of solving the U(1)-problem quantitatively using the $1/N_c$-expansion [30].

Let us discuss now the choice of the function $f(k^2)$ in (1) which is closely connected with the problem of describing the Faddeev–Popov ghosts. Examine the Schwinger–Dyson equation for the ghost propagator $S(p)$:

$$S^{-1}(p) - {S^{(0)}}^{-1}(p) = \frac{C_2}{(2\pi)^n} \int \mathrm{d}^n k \; D_{\mu\nu}(k) \; \Lambda_\mu^{(0)}(p, p-k; -k) \; S(p-k) \; \Lambda_\nu(p-k, p; k) \; , \quad (2)$$

where $S^{(0)}(p) = 1/p^2$ is a free (bare) ghost propagator; $\Lambda_\mu^{(0)} = g p_\mu$ and Λ_ν are, respectively, free and proper vertices "ghost-ghost-gluon"; $C_2 = N_c$ is a color factor. Here and henceforth, we use the dimensional regularization [31] ($n = 4 + 2\varepsilon$, $\varepsilon \to 0$). It has been shown in [32] that, if the Fourier transform of the gluon propagator $D(k) \sim M^2/(k^2)^2$ is transversal in the coordinate space,

$$\widetilde{D}_{\mu\nu}(x) \; x_\nu = 0 \; , \quad (3)$$

then the loop integral on the r.h.s. of (2) vanishes and, therefore, equation (2) in the infrared region is satisfied by free expressions for $S(p)$ and Λ_ν. In this case the description of ghost fields proves to be simplest. This result can readily be generalized to the case of an arbitrary form of the gluon propagator, namely, for the ghost propagator equation to have a free solution it is sufficient that the condition (3) be satisfied. In case, for example, of an arbitrary power-like behaviour

$$D_{\mu\nu}(k|\gamma) = \frac{(M^2)^{\gamma-1}}{(k^2)^\gamma} \left(g_{\mu\nu} - d(\gamma) \frac{k_\mu k_\nu}{k^2} \right)$$

the transversality condition in the x-space (3) leads to

$$d(\gamma) = \frac{2\gamma}{2\gamma + 1 - n} \qquad (n = 4 + 2\varepsilon) \; . \quad (4)$$

In the particular case $\gamma = 1$ (the ultraviolet region), equation (4) corresponds to the Soloviev–Yennie gauge [33],

$$d(1) = -2/(n-3) = -2 + 4\varepsilon + \mathcal{O}(\varepsilon^2) \; , \quad (5)$$

and to the Arbuzov gauge [34, 24]

$$d(2) = 4/(5-n) = 4 + 8\varepsilon + \mathcal{O}(\varepsilon^2) \tag{6}$$

at $\gamma = 2$ (the infrared region). It should be noted that the similar condition (3) is also satisfied by the propagator $< T(A_\nu(x)A_\mu(0)) >$ in the Fock–Schwinger gauge which does not contain any ghost contributions.

Thus, if we choose the function $f(k^2)$ to be of the following form at small and large k^2 values, respectively,

$$f^{(\mathrm{IR})}(k^2) = (1 - d(2))\frac{M^2}{(k^2)^2} \;; \quad f^{(\mathrm{UV})}(k^2) = (1 - d(1))\frac{1}{(k^2)} \;, \tag{7}$$

the description of ghosts will be simplest and, in particular, the contributions from ghosts will disappear (see [35]) in the Ward–Slavnov–Taylor identities for the gluon vertices.

As a result, we get the following expressions for the gluon propagator (1) in the infrared and ultraviolet regions:

$$D^{(\mathrm{IR})}_{\mu\nu}(k) = \frac{M^2}{(k^2)^2}\left(g_{\mu\nu} - d(2)\frac{k_\mu k_\nu}{k^2}\right) \;, \tag{8}$$

$$D^{(\mathrm{UV})}_{\mu\nu}(k) = \frac{1}{k^2}\left(g_{\mu\nu} - d(1)\frac{k_\mu k_\nu}{k^2}\right) \;, \tag{9}$$

where $d(1)$ and $d(2)$ are defined by formulae (5) and (6).

It should be noted that in ref. [23] the compatibility of the infrared asymptotics (8) and the corresponding gluon vertices with the Schwinger–Dyson equation for gluon propagator was studied. Consideration was given, in particular, to the most infrared-singular two-loop terms of the equation. The Arbuzov gauge (6) was shown to yield a self-consistent description of the lowest gluon and ghost Green's functions (in other cases, one has to allow for the non-trivial effect of ghosts on the gauge identities which are used to reconstruct the gluon vertices).

Concluding this Section, we shall mention the properties of the three-gluon vertex $\Gamma^{abc}_{\mu\nu\rho}(p,t,k) = gf^{abc}\Gamma_{\mu\nu\rho}(p,t,k)$ (f^{abc} are structure constants of $\mathrm{SU_c}(3)$ group) which will be used in the discussion below. In the Euclidean momentum range $p^2 \gg k_0^2$, $t^2 = (-p-k)^2 \gg k_0^2$ and $k^2 \ll k_0^2$, this vertex may be expanded in the small-to-large momentum ratio [18]. In this case, $\Gamma_{\mu\nu\rho}(p,-p-k,k) \simeq \Gamma_{\mu\nu\rho}(p,-p,0)$, up to the terms of higher order in k/p. The vertex $\Gamma_{\mu\nu\rho}(p,-p,0)$ is related through a gauge identity [36] to the gluon polarization operator, which is defined in the examined $p^2 \gg k_0^2$ range by the perturbation theory. In the general case, however, this identity does not reproduce the ordinary differential Ward identity, because it is affected nontrivially by the ghost Green's functions (see, e.g., [37]), in particular, by the ghost propagator treated in the non-perturbative region $k \to 0$. Only when the function $f(k^2)$ is chosen to be of the form (7), will not the ghost (as noted above) affect the gauge identity [35], so the latter will take the form of the differential Ward identity. In this case the vertex $\Gamma_{\mu\nu\rho}(p,-p,0)$ will be defined at $p^2 \gg k_0^2$ by the perturbation theory.

3. The Bethe–Salpeter equation and the glueball mass estimation

Let us examine now the Bethe–Salpeter equation for the wave function of bound state of two gluons in the Euclidean momentum space. The diagrammatic representation of the equation is shown in Fig. 1 where Φ and K are respectively the wave function and the Bethe–Salpeter kernel, η is transversal projector, Π is the full polarization operator which is related to the gluon propagator as

$$\Pi_{\mu\rho}(p)\ D_{\rho\nu}(p) = g_{\mu\nu} - p_\mu p_\nu / p^2 \equiv \eta_{\mu\nu}(p)\ .$$

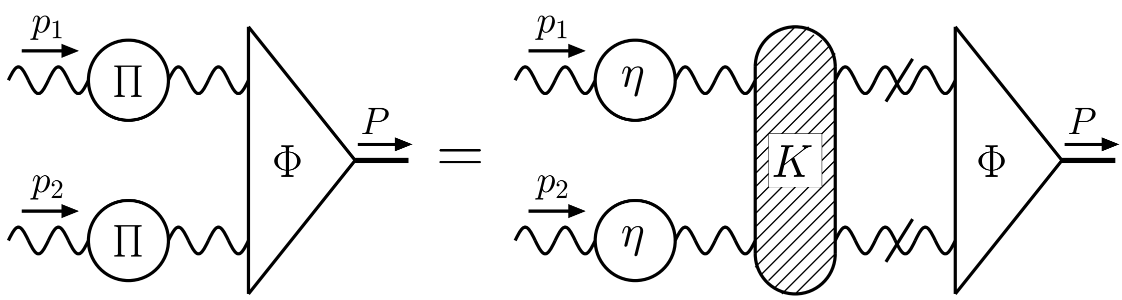

Fig. 1: Graphic representation of the Bethe–Salpeter equation for the wave function of bound state of two gluons

Basing on the reasoning of Section 1, we shall treat the Bethe–Salpeter equation in such kinematic region that the total momentum of the gluons $P^2 > k_0^2$ and the relative momentum $q^2 < k_0^2$. Let the loop integral over momentum k in the equation be divided into the "soft" and "hard" parts:

$$\int \mathrm{d}k = \int \mathrm{d}k\ \theta(k_0^2 - k^2) + \int \mathrm{d}k\ \theta(k^2 - k_0^2) \equiv \int\limits_{\text{(IR)}} \mathrm{d}k + \int\limits_{\text{(UV)}} \mathrm{d}k\ .$$

In the range of high virtualities $k^2 > k_0^2$ the kernel K is defined in the main approximation by the perturbation theory, thereby yielding an asymptotically-free negative potential (more strictly, a quasi-potential), and, as noted in Section 1, is irrelevant to glueball (at least, in the leading order). In the range of low virtualities $k^2 < k_0^2$ the main contribution to the kernel K appears to be given by the exchange by a single "infrared" gluon [18] (see Fig. 2) with the propagator $D(k) \sim M^2/(k^2)^2$ (since, in each of the three-gluon vertices, two momenta are hard and one is soft, these vertices are free in the leading approximation because of the property mentioned in Section 2).

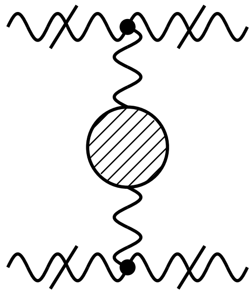

Fig. 2: The contribution of the exchange by a single "soft" gluon to the kernel of the Bethe–Salpeter equation

Indeed, it can readily be verified that all other possible contributions to the Bethe–Salpeter kernel appear to be suppressed compared with the one-gluon contribution, either by the running constant $\alpha_s(P^2)$ or by the parameter $k_0^2/P^2 \sim (gM/\pi)^2/P^2$ (in case the glueball mass is sufficiently high; it is this kinematic region that is examined here). It should be noted that only one of the two parameters k_0 and gM/π is independent (the relation between k_0 and gM/π is discussed in [19]), for example, gM/π whose value (0.59 GeV) is determined from the known slope of the linearly-rising potential. For instance, let us examine the contributions to the kernel shown in Fig. 3: the contribution of the "cross" exchange by two soft gluons (Fig. 3a) is suppressed by the parameter $(gM/\pi)^2/P^2$, whereas the contribution of the diagram shown in Fig. 3b with an additional hard gluon is suppressed by the running coupling constant $\alpha_s(P^2)$.

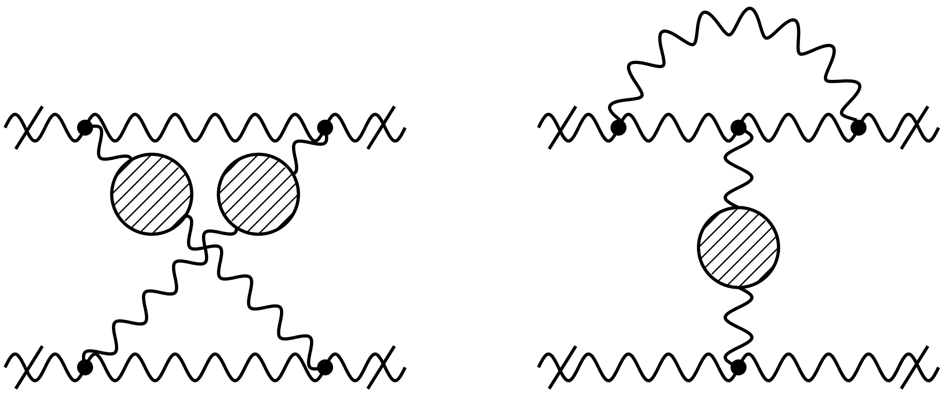

Fig. 3: Examples of other contributions to the Bethe–Salpeter kernel

Thus, in the examined approximation, the equation in Fig. 1 takes the form (summation over the color indices has been performed)

$$\Pi_{\mu\mu'}(p_1)\Pi_{\nu\nu'}(p_2)\Phi_{\mu'\nu'}(q,P) = -\frac{C_2 g^2}{(2\pi)^n}\eta_{\mu\mu'}(p_1)\eta_{\nu\nu'}(p_2) \int\limits_{(\mathrm{IR})} \mathrm{d}^n k \; D^{(\mathrm{IR})}_{\gamma\omega}(k)$$
$$\times\Gamma_{\mu'\rho\gamma}(-p_1, p_1 - k, k)\; \Gamma_{\nu'\sigma\omega}(-p_2, p_2 + k, -k)\; \Phi_{\rho\sigma}(q-k, P)\;, \qquad (10)$$

where $C_2 = N_c = 3$ is color factor; $q = (p_1 - p_2)/2$. Taking into account the above considerations and the above-mentioned property of three-gluon vertices, we shall expand the r.h.s. of (10) in the ratio of "small" momenta q and k to the "large" momentum P, obtaining

$$-\frac{3g^2}{(2\pi)^n}\,\eta_{\mu\mu'}(P)\,\eta_{\nu\nu'}(P)\,\Gamma^{(0)}_{\mu'\rho\gamma}\left(-\frac{P}{2},\frac{P}{2},0\right)\Gamma^{(0)}_{\nu'\sigma\omega}\left(-\frac{P}{2},\frac{P}{2},0\right)\times\Phi_{\rho\sigma}(0,P)\int\limits_{(\mathrm{IR})}\mathrm{d}^n k\;D^{(\mathrm{IR})}_{\gamma\omega}(k)\;. \tag{11}$$

The integral in (11) of the gluon propagator (8) in the applied Arbuzov gauge (6) is finite as $n \to 4$ and yields [18]

$$\int\limits_{(\mathrm{IR})}\mathrm{d}^n k\;D^{(\mathrm{IR})}_{\gamma\omega}(k) = -\frac{3}{2}\pi^2 M^2 g_{\gamma\omega}\;. \tag{12}$$

This integral is independent of the cutoff parameter k_0 because the main contribution to the integral comes from the infinitesimal vicinity of the singular point $k = 0$ (this fact supplies an additional argument for neglecting the $\mathcal{O}(k/P)$ terms in the integrand).

The tensor structure of the Bethe–Salpeter wave function Φ, which has the sence of the vertex of the interaction of gluons with glueball, may be extracted from considering the effective actions of the type [15]

$$A = \int \mathrm{d}x\;\mathrm{d}y\;\mathrm{d}z\;G^a_{\mu\alpha}(x)\;G^a_{\alpha\nu}(y)\;\varphi(z)\;\chi_{\mu\nu}(x,y|z)$$

in case of a scalar glueball. From this we obtain in case of 0^{++} glueball that

$$\Phi_{\rho\sigma}(q,P) = D_{\rho\mu}(P/2+q)\;D_{\sigma\nu}(P/2-q)\;\chi_{\mu\nu}(q;P)$$

has the structure (at $q = 0$)

$$\Phi_{\rho\sigma}(0,P) = \left(g_{\rho\sigma} - \frac{P_\rho P_\sigma}{P^2}\right)\Phi(0,P)\;. \tag{13}$$

Substituting (12) and (13) in (11), we get the following expression for the r.h.s. of the Bethe–Salpeter equation:

$$\frac{9}{32}\left(\frac{gM}{\pi}\right)^2\left(g_{\mu\nu}P^2 - P_\mu P_\nu\right)\Phi(0,P)\;. \tag{14}$$

Taking into account the above considerations, the l.h.s. of (10) takes the form

$$\Pi_{\mu\mu'}(P/2)\;\Pi_{\nu\nu'}(P/2)\;\left(g_{\mu'\nu'} - P_{\mu'}P_{\nu'}/P^2\right)\;\Phi(0,P)\;. \tag{15}$$

Apart from the free expressions,

$$\Pi^{(0)}_{\mu\nu}(p) = g_{\mu\nu}p^2 - p_\mu p_\nu\;, \tag{16}$$

the polarization operators Π contain non-perturbative corrections of the order $(gM/\pi)^2/P^2$ which must be included in our treatment (this was not done in [18]). The appearance of these corrections can readily be understood by examining the one-loop terms of the Schwinger–Dyson equation for the gluon polarization operator. In case the external momentum p is large, small momenta k corresponding to the infrared gluon propagator (8) can nevertheless run along individual gluon lines. The diagrammatic representation of the expressions for these corrections is shown in Fig. 4 where the shaded circles serve to distinguish the "infrared" gluon lines from "ultraviolet" ones. It should be noted that, in case the large momenta run along all the lines of the loop, the corresponding expression will be of the order $\alpha_{\mathrm{s}}(P^2)$, whereas the leading non-perturbative corrections are of the order $[(gM/\pi)^2/P^2]^2$, and are disregarded here.

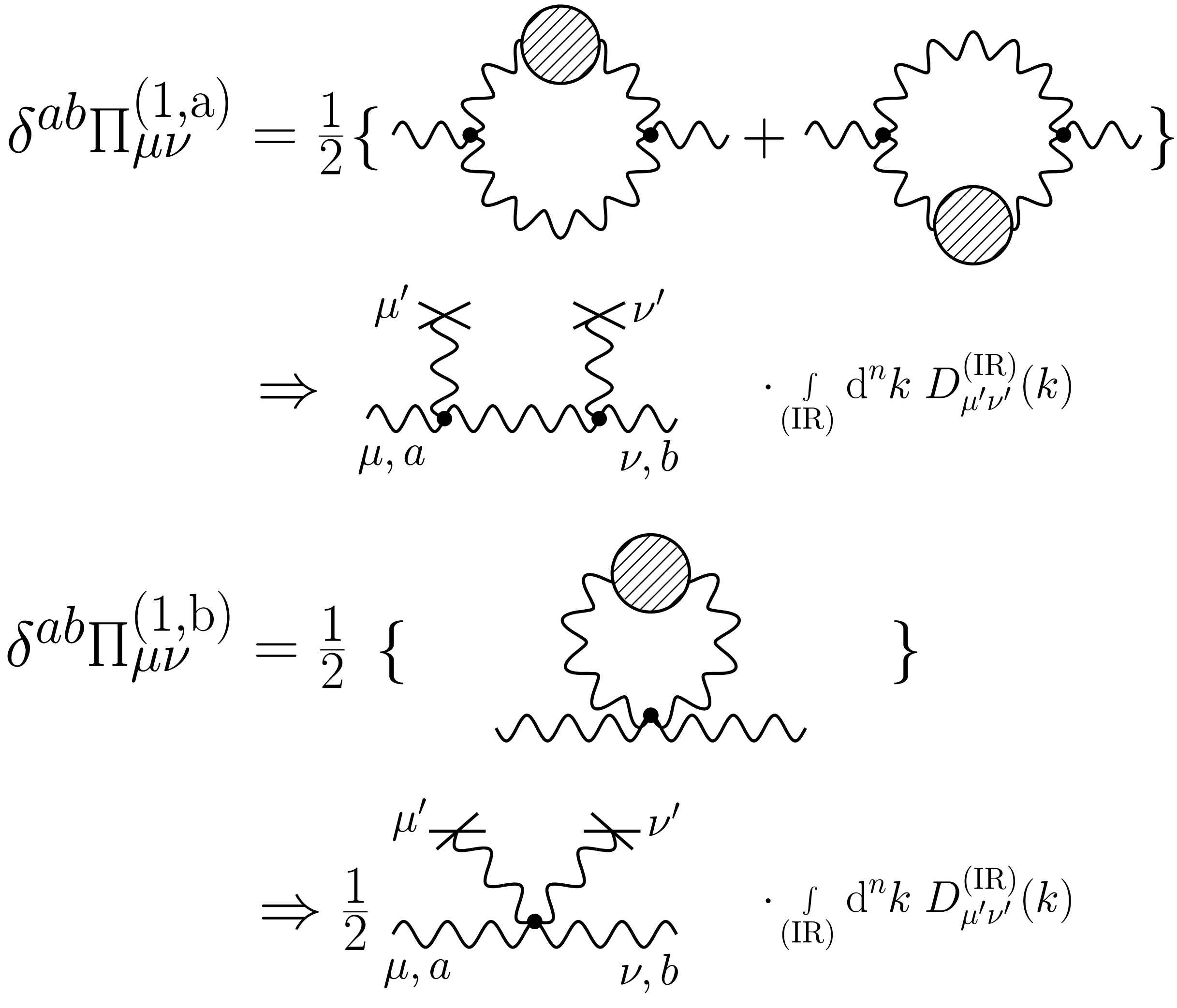

Fig. 4: The leading corrections to the gluon polarization operator

In our case, calculating the leading corrections (see Fig. 4) yields (allowing for (12)) the expressions

$$
\begin{aligned}
\Pi^{(1,\mathrm{a})}_{\mu\nu}(p) &= \frac{9}{32}\left(\frac{gM}{\pi}\right)^2\left[(5-d(1))g_{\mu\nu}-(2-d(1))\frac{p_\mu p_\nu}{p^2}\right] ,\\
\Pi^{(1,\mathrm{b})}_{\mu\nu}(p) &= \frac{9}{32}\left(\frac{gM}{\pi}\right)^2\left[-3g_{\mu\nu}\right] .
\end{aligned}
$$

As expected, their sum is transverse,

$$\Pi^{(1)}_{\mu\nu}(p) = \frac{9}{32}\left(\frac{gM}{\pi}\right)^2 (2 - d(1)) \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) , \tag{17}$$

where $d(1)$ corresponds to the Soloviev–Yennie gauge (5); $d(1) = -2$ at $n = 4$. Therefore,

$$\Pi^{(1)}_{\mu\nu}(p) = \frac{9}{8}\left(\frac{gM}{\pi}\right)^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) ,$$

and we obtain the following expression for the l.h.s. of (15):

$$\left[\frac{P^2}{16} + \frac{9}{16}\left(\frac{gM}{\pi}\right)^2\right] (g_{\mu\nu}P^2 - P_\mu P_\nu)\Phi(0, P) . \tag{18}$$

As a result, the examined equation (10) reduces to

$$\left[P^2 + \frac{9}{2}\left(\frac{gM}{\pi}\right)^2\right] \Phi(0, P) = 0 . \tag{19}$$

Equation (19) is essentially the Klein–Gordon equation with respect to the Euclidean momentum P^2 for a scalar glueball, whence we obtain the following value of the mass of the 0^{++} glueball:

$$M^2_{\rm gl} = \frac{9}{2}\left(\frac{gM}{\pi}\right)^2 . \tag{20}$$

Substituting the numerical value $gM/\pi \simeq 0.59$ GeV, we get

$$M_{\rm gl} \simeq 1.25 \ \ \text{GeV} . \tag{21}$$

All the above operations can also be performed for the case of a glueball with quantum numbers 2^{++}. In this case the corresponding state is singled out by the density matrix

$$T_{\mu\nu\alpha\beta}(P) = \eta_{\mu\alpha}(P)\eta_{\nu\beta}(P) + \eta_{\mu\beta}(P)\eta_{\nu\alpha}(P) - \frac{2}{3}\eta_{\mu\nu}(P)\eta_{\alpha\beta}(P) ,$$

and

$$\widetilde{\Phi}_{\mu\nu\alpha\beta}(0, P) = T_{\mu\nu\alpha\beta}(P)\widetilde{\Phi}(0, P) .$$

After the calculations, the same equation (19) proves again to be obtained for $\widetilde{\Phi}(0, P)$, so the masses of the states 0^{++} and 2^{++} appear to be degenerate in the examined approximation.

4. Conclusion

The estimate obtained for the 0^{++} glueball mass and the 0^{++} and 2^{++} mass degeneracy are in good agreement with the lattice calculation results. Listed below are the relevant values obtained elsewhere: $m(0^{++}) = 1.4 \pm 0.1$ GeV and $m(2^{++})/m(0^{++}) = 1 \pm 0.2$ [8]; $m(0^{++}) = 1.2$ GeV [9]; $m(0^{++}) = 1.3$ GeV [10]; $m(0^{++}) = 1.24 \pm 0.06$ GeV [11]. The value $m(2^{++})/m(0^{++}) \sim 1$ was also obtained in refs. [38]. The result of [16], $m(0^{++}) =$

1.32 ± 0.02 GeV, obtained by the Rayleigh–Ritz variational method, is also not too far from (21) (true, the "fantastic" accuracy of the value obtained is somewhat confusing). From some data on the available glueball candidates presented in Section 1 it is seen that our estimate does not contradict them either.

We have obtained the glueball mass to within a $\sim 30\%$ accuracy which is defined by the order of discarded terms, namely, $(gM/\pi)^2 \sim 0.3$. Such an accuracy corresponds to the accuracy of the estimates in the QCD sum rules method, whose spirit influenced the expansions in the Bethe–Salpeter equation made in terms of the examined approach. Calculating the next terms of the expansion in parameters $(gM/\pi)^2/P^2$ and $\alpha_s(P^2)$ would be of great importance, but the given problem requires much greater efforts and has not been resolved yet.

With a view to correct comparing with experimental data, the possibility of mixing the gluon and quark states with identical quantum numbers must of course be allowed for. We are planning to calculate the glueball decay widths, which are also very interesting in terms of experiment.

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