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An Approach to the Evaluation of Three- and Four-Point Ladder Diagrams

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Abstract

An approach to the calculation of ladder graphs with three and four external lines is considered (in the case of massless internal particles and arbitrary external momenta). Simple formulae for reducing four-point diagrams to three-point vertices are derived. Exact results for diagrams up to the two-loop level are obtained in terms of polylogarithms.

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1 Introduction

Attention to the problem of evaluating ladder graphs in quantum field theory has been paid for a long time (see, e.g., [1] and references therein). Recent development of modern accelerators also makes this problem to be important for calculation of radiative corrections (Bhabha scattering, etc.). Moreover, in some cases the contribution of such diagrams can dominate non-planar graphs. The case of massless internal propagators is of special interest, because often we are confronted either with really massless particles (photon, gluon) or with particles whose masses can be neglected (in high-energy processes). In a number of publications [2, 3, 4] the asymptotic behaviour of ladder diagrams has been examined for high energies and momentum transfer in leading logarithmic approximation.

In the present paper we propose an approach which makes it possible to obtain exact results for three- and four-point ladder diagrams with massless internal particles and arbitrary external momenta. This approach involves the following tools: (i) the Feynman parametric representation, (ii) the "uniqueness" conditions (see, e.g., in [5]) and (iii) Mellin-Barnes contour integrals. We shall consider only scalar diagrams (corresponding to massless ϕ^3 theory), because expressions occurring in realistic calculations can be reduced to such scalar integrals (see, e.g., [6]). Although one- and two-loop diagrams only are considered in the present paper, the approach can also be applied to ladder graphs with arbitrary number of rungs.

The remainder of the paper is organized as follows. In Section 2 we present the main steps of the approach via examples of one-loop triangle and box diagrams. In Section 3 we use this method to calculate two-loop three- and four-point ladder diagrams. Section 4 formulates and discusses the main results of the paper.

2 One-loop diagrams

Let us start by some definitions and results for massless triangle diagrams (see Figure 1). Here and below we shall consider all external momenta to be ingoing ($p_1 + p_2 + p_3 = 0$). The corresponding Feynman integral is

$$J(n;\ \nu_1, \nu_2, \nu_3) \equiv \int \frac{d^n k}{((q_1+k)^2)^{\nu_1}((q_2+k)^2)^{\nu_2}((q_3+k)^2)^{\nu_3}} \tag{2.1}$$

where $q_3 - q_2 = p_1, q_1 - q_3 = p_2, q_2 - q_1 = p_3$, and n is the space-time dimension. The usual "causal" prescription for singularities in pseudo-Euclidean momentum space is understood,

$$((q+k)^2)^{-\nu} \leftrightarrow ((q+k)^2 + i0)^{-\nu}. \tag{2.2}$$

Introducing Feynman parameters we get

$$J(n;\ \nu_1, \nu_2, \nu_3) = \pi^{n/2} i^{1-n} \frac{\Gamma\left(\sum \nu_i - n/2\right)}{\prod \Gamma(\nu_i)} \int_0^1\!\!\int_0^1\!\!\int_0^1 \frac{\prod \alpha_i^{\nu_i - 1} d\alpha_i \ \ \delta\left(\sum \alpha_i - 1\right)}{\left[\alpha_1\alpha_2 p_3^2 + \alpha_1\alpha_3 p_2^2 + \alpha_2\alpha_3 p_1^2\right]^{\Sigma\nu_i - n/2}}. \tag{2.3}$$

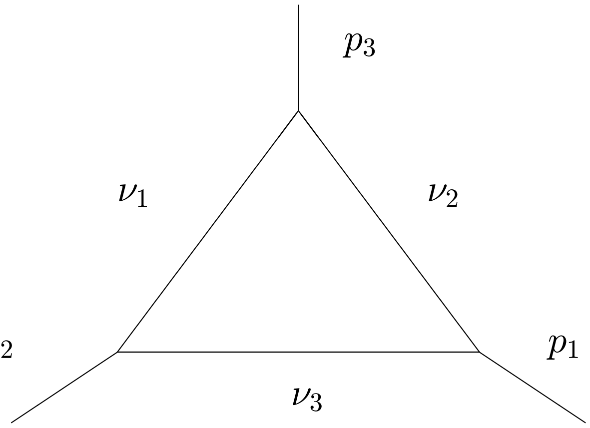

Fig. 1.

The integral (2.1) can also be represented in terms of a two-fold Mellin-Barnes integral [7, 8]

$$J(n;\ \nu_1,\nu_2,\nu_3) = \frac{\pi^{n/2}\ i^{1-n}\ (p_3^2)^{n/2-\Sigma\nu_i}}{\Gamma\left(n-\sum\nu_i\right)\ \prod\Gamma(\nu_i)}\frac{1}{(2\pi i)^2}\int\limits_{-i\infty}^{i\infty}\int\limits_{-i\infty}^{i\infty} du\ dv\ x^u\ y^v\ \Gamma(-u)\Gamma(-v)$$

$$\times\Gamma(n/2-\nu_2-\nu_3-u)\Gamma(n/2-\nu_1-\nu_3-v)\Gamma(\nu_3+u+v)\Gamma(\nu_1+\nu_2+\nu_3-n/2+u+v), \tag{2.4}$$

where

$$x \equiv \frac{p_1^2}{p_3^2} \quad , \quad y \equiv \frac{p_2^2}{p_3^2}\ . \tag{2.5}$$

and (here and below) the integration contours are chosen so as to separate the "right" and "left" series of poles of gamma functions in the integrand (see, e.g., [9]). By use of the resudue theorem, the result for arbitrary n and ν_i can be found in terms of hypergeometric functions of two variables (see [10, 7, 8]).

When the powers of denominators and n are related by $\nu_1+\nu_2+\nu_3=n$, a very simple result can be obtained from (2.3) for such a "unique" triangle [11, 12]:

$$J(n;\ \nu_1,\nu_2,\nu_3)\Big|_{\Sigma\nu_i=n} = \pi^{n/2}\ i^{1-n}\prod_{i=1}^{3}\frac{\Gamma(n/2-\nu_i)}{\Gamma(\nu_i)}(p_i^2)^{\nu_i-n/2}. \tag{2.6}$$

If the sum of the powers of denominators is $\nu_1+\nu_2+\nu_3=n-1$, then the following identity [5] (see also [13, 14]) holds for such "semi-unique" triangles (we write it in the momentum space):

$$\left\{\ \nu_1 J(n;\ \nu_1+1,\nu_2,\nu_3)+\nu_2 J(n;\ \nu_1,\nu_2+1,\nu_3)+\nu_3 J(n;\ \nu_1,\nu_2,\nu_3+1)\right\}\Big|_{\Sigma\nu_i=n-2}$$

$$= \pi^{n/2}\ i^{1-n}\prod_{i=1}^{3}\frac{\Gamma(n/2-\nu_i-1)}{\Gamma(\nu_i)}(p_i^2)^{\nu_i-n/2+1}. \tag{2.7}$$

This relation can also easily be derived by using (2.3). We shall use so-called "uniqueness" conditions (2.6) and (2.7) below, when evaluating two-loop diagram.

In applications the case $\nu_1 = \nu_2 = \nu_3 = 1,\ n = 4$ is most important. Let us denote

$$C^{(1)}(p_1^2, p_2^2, p_3^2) \equiv J(4; 1, 1, 1). \tag{2.8}$$

Then from (2.4) it is easy to find that (see [15])

$$C^{(1)} = \frac{i\pi^2}{p_3^2} \frac{1}{(2\pi i)^2} \int\limits_{-i\infty}^{i\infty} \int\limits_{-i\infty}^{i\infty} du\, dv\, x^u\, y^v\, \Gamma^2(-u)\Gamma^2(-v)\Gamma^2(1+u+v), \tag{2.9}$$

where x and y are defined in (2.5). Using (2.3) (or (2.9)) one can arrive at the following result:

$$C^{(1)}(p_1^2, p_2^2, p_3^2) = \frac{i\pi^2}{p_3^2} \Phi(x, y) \tag{2.10}$$

with

$$\Phi(x, y) = \frac{1}{\lambda} \left\{ 2\left(\mathrm{Li}_2\left(-\rho x\right) + \mathrm{Li}_2\left(-\rho y\right)\right) + \ln(\rho x)\ln(\rho y) + \ln\frac{y}{x} \ln\frac{1+\rho y}{1+\rho x} + \frac{\pi^2}{3} \right\}, \tag{2.11}$$

where $\mathrm{Li}_2\,(z)$ is Euler's dilogarithm (see, e.g., [16]) and

$$\lambda(x, y) \equiv \sqrt{(1-x-y)^2 - 4xy}, \tag{2.12}$$

$$\rho(x, y) \equiv \frac{2}{1-x-y+\lambda} \tag{2.13}$$

Note that representations of such type are well known for triangle diagrams (see, e.g., [17]). By simple dilogarithm transformations, formula (2.11) can be turned into the result obtained in [8]. In the paper [18] some problems of analytic continuation of the function $\Phi(x, y)$ (2.11) in the region of positive x and y were examined. If we consider, for example, negative values of both variables x and y, we should take into account the prescription (2.2). This requires the following substitutions in (2.11):

$$\ln(\rho x) \to \ln(-\rho x) + i\pi\sigma\ ; \qquad \ln(\rho y) \to \ln(-\rho y) + i\pi\sigma\ , \tag{2.14}$$

where $\sigma \equiv \mathrm{sgn}\, p_3^2$.

Let us consider now a four-point "box" diagram in Figure 2. All external momenta are taken to be ingoing ($k_1 + k_2 + k_3 + k_4 = 0$). We use standard notation

$$s \equiv (k_1 + k_2)^2\ , \quad t \equiv (k_2 + k_3)^2. \tag{2.15}$$

Then Feynman parametrization of the corresponding integral (with unit powers of denominators and $n = 4$) gives

$$\begin{aligned} &D^{(1)}(k_1^2, k_2^2, k_3^2, k_4^2, s, t) \\ &= i\pi^2 \int\limits_0^1 \ldots \int\limits_0^1 \frac{\prod d\alpha_i\ \ \delta\left(\sum \alpha_i - 1\right)}{\left[\alpha_1\alpha_2 k_1^2 + \alpha_2\alpha_3 k_2^2 + \alpha_3\alpha_4 k_3^2 + \alpha_1\alpha_4 k_4^2 + \alpha_1\alpha_3 s + \alpha_2\alpha_4 t\right]^2}. \end{aligned} \tag{2.16}$$

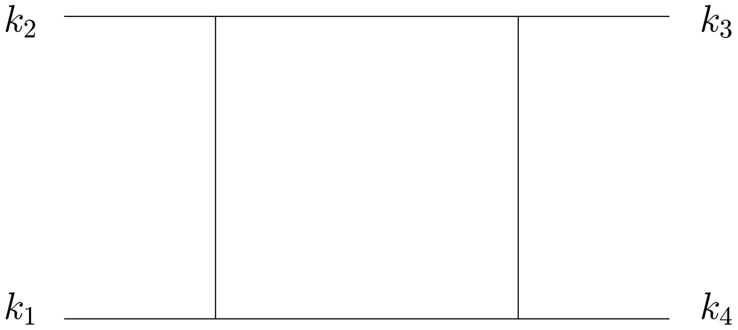

Fig. 2.

Substituting $\alpha_4 = \alpha,\ \alpha_i = (1-\alpha)\beta_i\ (i=1,2,3)$ and integrating over α, we get

$$D^{(1)} = i\pi^2 \int_0^1 \int_0^1 \int_0^1 \frac{\prod d\beta_i \ \ \delta\left(\sum \beta_i - 1\right)}{[\beta_1 k_4^2 + \beta_2 t + \beta_3 k_3^2]\,[\beta_1\beta_2 k_1^2 + \beta_1\beta_3 s + \beta_2\beta_3 k_2^2]}. \qquad (2.17)$$

Representing the first denominator in (2.17) through a two-fold Mellin-Barnes integral (see, e.g., [19, 20]),

$$\left[\beta_1 k_4^2 + \beta_2 t + \beta_3 k_3^2\right]^{-1}$$
$$= \frac{1}{\beta_2 t}\frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty} du\ dv \left(\frac{\beta_3 k_3^2}{\beta_2 t}\right)^u \left(\frac{\beta_1 k_4^2}{\beta_2 t}\right)^v \Gamma(-u)\Gamma(-v)\Gamma(1+u+v), \quad (2.18)$$

we see that the integration over β's yields an "unique" triangle $J(2;\ 1+v, -u-v, 1+u)$ with external invariants k_1^2, s, k_2^2. Using the identity (2.6) we find

$$D^{(1)} = \frac{i\pi^2}{s\ t}\frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty} du\ dv\ X^u\ Y^v\ \Gamma^2(-u)\Gamma^2(-v)\Gamma^2(1+u+v), \qquad (2.19)$$

where

$$X \equiv \frac{k_1^2 k_3^2}{st}, \quad Y \equiv \frac{k_2^2 k_4^2}{st}. \qquad (2.20)$$

Comparison with the representation (2.9) gives

$$D^{(1)}(k_1^2, k_2^2, k_3^2, k_4^2, s, t) = C^{(1)}(k_1^2 k_3^2, k_2^2 k_4^2, st), \qquad (2.21)$$

or

$$D^{(1)}(k_1^2, k_2^2, k_3^2, k_4^2, s, t) = \frac{i\pi^2}{s\ t}\Phi(X, Y), \qquad (2.22)$$

with the same function Φ as for triangle diagram. Thus, the expression obtained for the "box" diagram contains only two dilogarithms. For negative x and y (this

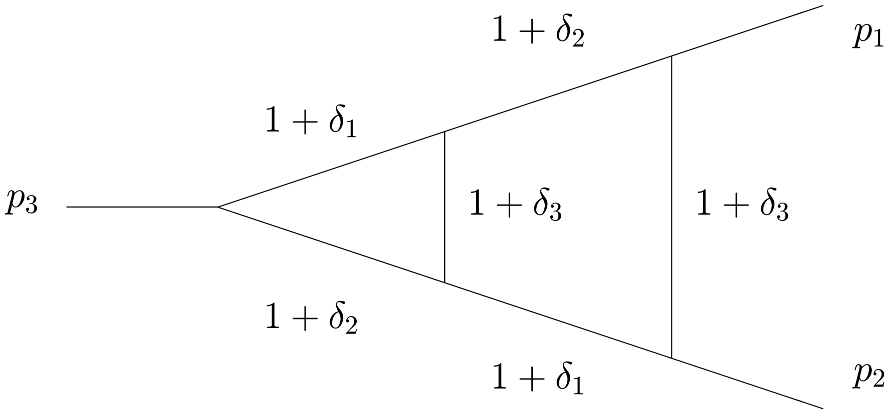

Fig. 3.

corresponds, e.g., to a physically interesting case when s and k_i^2 are positive while t is negative), it is necessary to use a prescription of the type of (2.14) (for details see in [21]). Note that our expression is more compact than one obtained in [21] for the special case when the masses vanish (the result of [21] involves four dilogarithms and can be reduced to our expression by use of simple relations for dilogarithms).

We note a useful fact of "pairing" of variables in the four-point function (2.21) allowing reduction to the three-point function. Below we shall see that an analogous property also occurs for two-loop diagrams.

3 Two-loop diagrams

Let us consider first a three-point two-loop ladder (planar) diagram shown in Figure 3 ($p_1 + p_2 + p_3 = 0$). The Feynman integral corresponding to this diagram (with unit powers of denominators) can be written as

$$C^{(2)}(p_1^2, p_2^2, p_3^2) = \int \frac{d^4 r}{r^2 (p_1+r)^2 (p_2-r)^2} C^{(1)}((p_1+r)^2, (p_2-r)^2, p_3^2), \tag{3.1}$$

where $C^{(1)}$ is the one-loop function (2.8). If we use the representation (2.9) for $C^{(1)}$ and represent the obtained integral over r in terms of contour integrals (2.4), then we get a four-fold Mellin-Barnes integral,

$$C^{(2)} = \left(\frac{i\pi^2}{p_3^2}\right)^2 \frac{1}{(2\pi i)^4} \int\limits_{-i\infty}^{i\infty} \ldots \int\limits_{-i\infty}^{i\infty} du\; dv\; du'\; dv'\; x^u\; y^v\; \Gamma(-u)\Gamma(-v) \frac{\Gamma(-u')\Gamma(-v')}{u'\; v'}$$
$$\times \Gamma(u'-u)\Gamma(v'-v)\Gamma(1+u+v)\Gamma(1+u'+v')\Gamma(1+u+v-u'-v'), \tag{3.2}$$

where x and y are defined by (2.5). We shall employ the representation (3.2) below, when studying the four-point diagram.

To calculate $C^{(2)}$, it is convenient to use the "uniqueness" method, by analogy with the paper [13] (where propagator-type ladder diagrams have been examined). To do this, let us consider a special analytic regularization of this diagram (see Fig.3), where we replace unit powers of denominators by $(1+\delta_i)$, provided that $\delta_1+\delta_2+\delta_3=0$. Applying relations (2.6) and (2.7) to this regularized diagram gives the following result (at $n=4$):

$$\frac{i\pi^2}{(p_3^2)^{1-\delta_3}}\prod\frac{\Gamma(1-\delta_i)}{\Gamma(1+\delta_i)}\left\{\frac{1}{\delta_1\delta_2}J(4;\ 1,1,1+\delta_3)\right.$$
$$\left.+\frac{1}{\delta_1\delta_3}(p_1^2)^{\delta_1}J(4;\ 1,1,1-\delta_2)+\frac{1}{\delta_2\delta_3}(p_2^2)^{\delta_2}J(4;\ 1,1,1-\delta_1)\right\}, \qquad (3.3)$$

where one-loop integrals J are defined in (2.1). Note that integrals on the r.h.s. of (3.3) can be transformed by use of the formulae

$$J(4;\ 1,1,1+\delta)=(p_1^2)^{-\delta}J(4;\ 1+\delta,1-\delta,1)=(p_2^2)^{-\delta}J(4;\ 1-\delta,1+\delta,1). \qquad (3.4)$$

These relations can also be obtained from (2.6) and (2.7).

Let us define a dimensionless function $\Phi(x,y|\delta)$ by

$$J(4;\ 1,1,1+\delta)\equiv\frac{i\pi^2}{(p_3^2)^{1+\delta}}\Phi(x,y|\delta), \qquad (3.5)$$

where x and y, as usual, are defined by (2.5). Note that the function $\Phi(x,y|\delta)$ is symmetric with respect to x and y, and at $\delta=0$ it coincides with $\Phi(x,y)$ (2.10)-(2.11). Below we shall not write arguments of the function Φ and its derivatives, if they are taken at $\delta=0$. Then the expansion in δ near $\delta=0$ takes the form ($\partial_\delta\equiv\partial/\partial\delta$):

$$\Phi(x,y|\delta)=\Phi+\delta\ \partial_\delta\Phi+\frac{1}{2}\delta^2\ \partial_\delta^2\Phi+... \qquad (3.6)$$

Using (3.4) it is easy to show that only "even" derivatives are independent in this expansion. For example, one can find for the first derivative (at $\delta=0$) that

$$\partial_\delta\Phi=-\frac{1}{2}\ \ln(xy)\ \Phi. \qquad (3.7)$$

Using (3.5)-(3.7), from (3.3) we obtain (as $\delta_i\to 0,\sum\delta_i=0$) :

$$C^{(2)}(p_1^2,p_2^2,p_3^2)=\frac{1}{2}\left(\frac{i\pi^2}{p_3^2}\right)^2\left\{3\partial_\delta^2\Phi-(\ln^2x+\ln^2y+\ln x\ln y)\Phi\right\}. \qquad (3.8)$$

Note that formulae (3.3) and (3.8) yield a representation of two-loop diagram in terms of the function connected with one-loop diagram. The results in terms of hypergeometric series [10, 7, 8] give asymptotic expansion of (3.8) for small values of x and y (with logarithmic terms).

By use of (3.4) and Feynman parameters (2.3), after some transformations one can obtain the following representation:

$$\Phi(x,y|\delta)=\frac{1}{\delta}\int\limits_0^1 d\xi\frac{(y\xi)^{-\delta}-(x/\xi)^{-\delta}}{y\xi^2+(1-x-y)\xi+x}. \qquad (3.9)$$

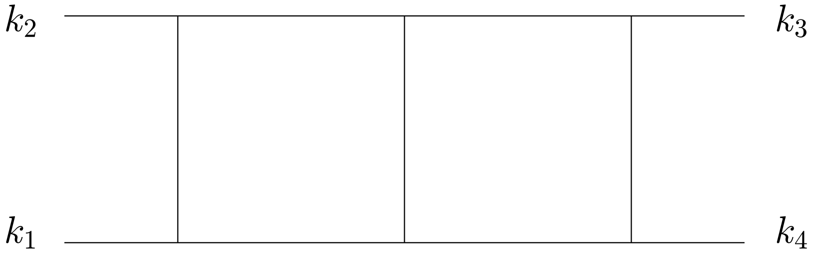

Fig. 4.

Inserting (3.9) into (3.8) we get

$$C^{(2)} = -\left(\frac{i\pi^2}{p_3^2}\right)^2 \int\limits_0^1 \frac{d\xi}{y\xi^2 + (1-x-y)\xi + x} \left\{ \ln^3\xi + \frac{3}{2}\ln\frac{y}{x}\ln^2\xi + \frac{1}{2}\ln^2\frac{y}{x}\ln\xi \right\}. \tag{3.10}$$

This integral can be easily calculated in terms of polylogarithms $\mathrm{Li}_N\,(z)$ (see [16]),

$$\mathrm{Li}_N\,(z) = \frac{(-1)^N}{(N-1)!}\int\limits_0^1 d\xi \frac{\ln^{N-1}\xi}{\xi - z^{-1}}. \tag{3.11}$$

So, we arrive at the following result for the two-loop three-point diagram of Fig. 3:

$$\begin{aligned} C^{(2)} = -\left(\frac{i\pi^2}{p_3^2}\right)^2 \frac{1}{\lambda} \Big\{ & 6\left(\mathrm{Li}_4\,(-\rho x) + \mathrm{Li}_4\,(-\rho y)\right) + 3\ln\frac{y}{x}\left(\mathrm{Li}_3\,(-\rho x) - \mathrm{Li}_3\,(-\rho y)\right) \\ & + \frac{1}{2}\ln^2\frac{y}{x}\left(\mathrm{Li}_2\,(-\rho x) + \mathrm{Li}_2\,(-\rho y)\right) + \frac{1}{4}\ln^2(\rho x)\ln^2(\rho y) \\ & + \frac{\pi^2}{2}\ln(\rho x)\ln(\rho y) + \frac{\pi^2}{12}\ln^2\frac{y}{x} + \frac{7\pi^4}{60} \Big\}, \end{aligned} \tag{3.12}$$

where $\lambda(x,y)$ and $\rho(x,y)$ are defined in (2.12) and (2.13), respectively. For negative values of x and y, one has to use (2.14).

Let us consider now the two-loop four-point diagram (”double box”) presented in Figure 4. All notations correspond to the one-loop case (in particular, s and t are defined by (2.15)). The corresponding integral can be represented as

$$\begin{aligned} & D^{(2)}(k_1^2, k_2^2, k_3^2, k_4^2, s, t) \\ & = \int \frac{d^4 r}{r^2 (k_3+r)^2 (k_4-r)^2} D^{(1)}(k_1^2, k_2^2, (k_3+r)^2, (k_4-r)^2, s, (k_2+k_3+r)^2), \end{aligned} \tag{3.13}$$

where $D^{(1)}$ is one-loop function (2.16). Reducing $D^{(1)}$ to the triangle diagram (2.21) and using the appropriate Mellin-Barnes representation (2.19), we see that the resulting integral over r corresponds to one-loop ”box” with shifted powers of denominators.

As a result, we get additional factors $\alpha_1^{-u}\alpha_2^{u+v}\alpha_3^{-v}$ in the numerator of Feynman parametric representation (of the type of (2.16)). Then the obtained integral over α's can be evaluated in the same way as in the case of $D^{(1)}$ (2.17)-(2.19) (using the relation (2.6)). Comparing the resulting four-fold Mellin-Barnes representation of $D^{(2)}$ with that of $C^{(2)}$, we find

$$D^{(2)}(k_1^2, k_2^2, k_3^2, k_4^2, s, t) = t\, C^{(2)}(k_1^2\, k_3^2,\ k_2^2\, k_4^2,\ s\, t). \tag{3.14}$$

We see that in two-loop case we also have obtained a "pairing" of four-point function arguments. As a result, $D^{(2)}$ can be reduced to three-point function $C^{(2)}$ (3.12) (where we should make the substitutions $p_3^2 \to st,\ x \to X,\ y \to Y$; see (2.20)). Thus, formula (3.12) (combined with (3.14) and (2.20)) yields a representation of the "double box" $D^{(2)}$ (Fig.4) in terms of polylogarithms (3.11).

Note that the s-channel of the diagram in Fig.4 corresponds to a "horizontal double box" with initial particles momenta k_1 and k_2. On the other hand, the t-channel corresponds to a "vertical double box" with initial momenta k_2 and k_3.

4 Conclusion

In the present paper we considered (via one- and two-loop examples) an approach to the evaluation of ladder diagrams with three and four external lines with arbitrary momenta. We used Feynman parametrization (2.3), "uniqueness" conditions (2.6)-(2.7) and Mellin-Barnes contour integrals. It is shown that the corresponding four-point functions can be reduced to three-point ones (see (2.21) and (3.14)). Note that analogous formulae also occur for ladder diagrams with arbitrary number of rungs.

The results (2.11) and (3.12) are presented in terms of polylogarithms with simple arguments (it is convenient to use such expressions in realistic calculations). Note that applying relations (2.6) and (2.7) makes it possible to consider ladder diagrams with larger number of loops. One can show that they can be represented in terms of higher derivatives of $\Phi(x, y|\delta)$ (3.5) with respect to δ, which also can be expressed in terms of polylogarithms (3.11) (for example, a three-loop diagram will contain derivative $\partial_\delta^4\ \Phi$ and hence Li_6). In the general case of L-loop ladder diagram, we expect that the highest order of occurring polylogarithms will be equal to $2L$.

It should be noted that, if external momenta vanish, we get infrared (on-shell) singularities. For example, in the papers [22, 23, 24] the diagram of Fig.3 has been considered at $p_1^2 = p_2^2 = 0$ by use of dimensional regularization, and singularities have appeared as the poles in $\varepsilon = (4-n)/2$. In our (four-dimensional) approach in this case we can put $p_{1,2}^2 = \mu^2$ ($\mu \to 0$), and the singularities will be manifested as powers of $\ln \mu$. The same situation occurs with the limit of large s and $|t|$.

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